

Bailout Stigma

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Abstract

We develop a model of bailout stigma where accepting bailout signals a firm's balance-sheet weakness and worsens its funding prospect. To avoid stigma, a firm with high-quality legacy assets either withdraws from subsequent financing after receiving a bailout or refuses a bailout altogether to send a favorable signal. The former leads to a short-lived stimulation with subsequent market freeze even worse than if there were no bailouts. The latter revives the funding market, albeit with delay, to the level achievable without any stigma. Strikingly, a bailout offer is most effective when many firms reject it (to build a favorable reputation) rather than accept it.

Keywords: Adverse selection, bailout stigma, shorted-lived and delayed stimulation

JEL Codes: D82, G01, G18

1 Introduction

History is fraught with financial crises and large-scale government interventions, the latter often involving a highly visible and significant wealth transfer from taxpayers to banks and their creditors. According to an IMF estimate based on 124 systemic banking crises from around the world during the period 1970-2007, the average fiscal costs associated with crisis management were around 13 percent of GDP ([Laeven and Valencia, 2008](#)). More recently, during the 2007-2009 Great Recession, the US government paid \$125 billion for assets worth

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\$86-109 billion to the nine largest banks under the Troubled Asset Relief Program (TARP) (Veronesi and Zingales, 2010).¹ At the time of writing this paper, an unprecedented scale of bailouts is being organized to meet the challenges of the unfolding COVID-19 crisis. The benefits of such interventions are difficult to measure since they depend on the unobservable counterfactual that would have played out in the absence of such interventions.

Philippon and Skreta (2012) and Tirole (2012) develop a plausible counterfactual of a market freeze and a rationale for government interventions. The essence of the argument is that the government can jump-start a market that would otherwise freeze due to adverse selection. By cleaning up bad assets, or “dregs skimming,” through public bailouts, the government can improve market confidence, thereby galvanizing transactions in healthier assets. However, the flip side of such dregs-skimming is that bailouts can attach stigma to their recipients, and thus increase future borrowing costs. The fear of this stigma may in turn discourage financially distressed firms from accepting bailout offers in the first place.

Policy makers during the Great Recession were well aware of such a fear and took efforts to alleviate the stigma. At the now-famous meeting held on October 13, 2008, Henry Paulson, then Secretary of the Treasury, “compelled” the CEOs of the nine largest banks to be the initial participants in the TARP, precisely to eliminate the stigma (“Eight days: the battle to save the American financial system,” *The New Yorker*, September 21, 2009). The rates at the Fed’s discount window, usually set above the federal funds rate, were cut half a percentage point to counteract the stigma that using the window would signal distress (Geithner, 2015, p. 129).²

Despite these efforts, the stigma remained real and significant. Armantier et al. (2015) documented that the banks were willing to pay 44 basis points (bps) more for borrowing from the Term Auction Facility (TAF) than they would pay for using the discount window. Gauthier et al. (2015) further demonstrate that the banks that accessed the TAF in 2008 paid approximately 31 bps less in interbank lending in 2010 than those that used the discount window. Given that the TAF was designed to hide the identities of its users, a possible explanation is that banks wanted to avoid stigma attached to using the discount window. There are also anecdotes highlighting the presence of stigma. Ford refused rescue loans under the Auto Industry Program in the TARP, with a view to “legitimately portraying itself as the health-

¹Congressional Budget Office (2012) estimates the overall cost of the TARP at approximately \$32 billion, the largest part of which stems from assistance to AIG and the automotive industry while capital injections to financial institutions are estimated to have yielded a net gain. For detailed assessments of the various programs in the TARP, see the *Journal of Economic Perspectives* (2015). See also Fleming (2012) who discusses how the various emergency liquidity facilities provided by the Federal Reserve during the 2007-2009 crisis were designed to overcome the limitations of traditional policy instruments at the time of crisis. Tong and Wei (2020) provide international evidence on the effect of unconventional interventions during 2008-2010.

²Such a concern is echoed in a speech given by the former Federal Reserve chairman Ben Bernanke in 2009: “The banks’ concern was that their recourse to the discount window, if it became known, might lead market participants to infer weakness—the so-called stigma problem.”

iest of Detroit’s automakers” (“A risk for Ford in shunning bailout, and possibly a reward,” *The New York Times*, December 19, 2008).³ Such reluctance to receive government offers of recapitalization was also noted during the Japanese banking crisis of the 1990s (Corbett and Mitchell, 2000; Hoshi and Kashyap, 2010). While differing in nature, a similar concern about bailout stigma is expressed in the wake of the current COVID-19 crisis (“Bernanke and Yellen, the Federal Reserve must reduce long-term damage from coronavirus,” *The Financial Times*, March 23, 2020).

Examples such as the above raise questions about whether public bailouts are effective and, if so, how they should be designed in light of the associated stigma. We address these questions by analyzing a two-period model of bailouts that can address reputational concerns most parsimoniously. There is a continuum of firms, each with one unit of a legacy asset in each period. The quality of asset in both periods is identical for each firm and is its private information unobserved by other parties. In each period, firms have access to profitable investment opportunities. However, the liquidity constraint and the lack of pledgeability of projects imply that firms need to sell assets in the market to fund those projects. As in Tirole (2012), we focus on the case where adverse selection leads to a market freeze and calls for a public bailout. To understand the reputational consequence of accepting a bailout, we assume the government runs a bailout program in the first period only, and firms must sell their assets in the market to fund their projects in the second period (as well as in the first period). The sale of assets in the market is not publicly observed, but firms’ acceptance of the bailout offer is. The market updates its belief on the quality of assets based on the observation of firms’ decisions on asset sale made in the first period, and thus makes its second-period offer accordingly.

Bailout stigma is captured in our model by the deterioration of terms that bailout recipients experience on the sale of their assets in the second period. As shown by Philippon and Skreta (2012) and Tirole (2012), a key function of public bailouts is dregs-skimming: by taking out the left tail of the worst quality assets, the bailout improves the perceived quality of remaining assets, thereby rejuvenating asset trade in the market. But the dregs-skimming through

³The market initially perceived Ford’s refusal to accept a bailout as a risky move, which was reflected in the rise in Ford’s CDS spreads relative to Chrysler’s. However, Ford’s profit and stock price showed a remarkable turnaround in 2009, part of which is attributed to the respect Ford garnered with customers and investors by refusing a bailout. (<http://www.nasdaq.com/investing/ford-turns-a-profit-after-turning-down-bailout.aspx>, accessed Nov 17, 2015). In a similar vein, participants in the TARP were eager to exit the program early, often citing stigma as their main motivation. Signature Bank of New York was one of the first to repay its TARP debt of \$120 million for this reason. Its chairman, Scott A. Shay, said, “We don’t want to be touched by the stigma attached to firms that had taken money.” (“Four small banks are the first to pay back TARP funds,” *The New York Times*, April 1, 2009). It is also well known that Jamie Dimon, CEO of JP Morgan Chase, wanted to exit TARP to avoid the stigma (“Dimon says he’s eager to repay ‘Scarlet Letter’ TARP,” *Bloomberg*, April 16, 2009). Of course, the fear of stigma is not the only reason for an early exit. Wilson and Wu (2012) find that early exit by banks is also related to CEO pay, bank size, capital, and other financial conditions.

bailouts implies that the market is more likely to believe those that accept the bailout to be less investment-worthy than those that refuse the bailout. Such a belief is reflected in the differential treatment of the two groups of firms in the second period: the market’s offer to bailout recipients would be in general in worse terms than that to the bailout holdouts.

The precise mechanism of how the bailout stigma affects the efficacy of the bailout crucially depends on firms’ strategic responses to the bailout offer. We identify two equilibrium outcomes—**short-lived stimulation** and **delayed stimulation**.

A short-lived stimulation equilibrium arises when high-quality firms strategically avoid the bailout stigma by accepting a bailout in the first period but withdrawing from the market in the second period. Since bailout recipients with high-quality assets are more likely to withdraw from the market—a simple consequence of the single-crossing property—, those that accept a bailout but are compelled to participate in the second period market suffer a severe stigma. To avoid that stigma, in the first period firms would rather fund their projects by selling their assets to the market at discount. This in turn deteriorates their sale terms, ultimately inflicting a commensurate haircut on the market-sellers; in effect, bailout stigma has “spread” from bailout recipients to market sellers. This contagion of stigma in turn leads firms with high-quality assets to avoid selling to the market, opting instead to accept bailout offers but withdraw from the second-period market/financing altogether to avoid that stigma.⁴ Finally, their withdrawal from the second-period market inflicts a severe stigma on those firms (with low-quality assets) participating in that market, thus completing the feedback loop.

The presence of firms that accept a bailout but withdraw from the subsequent market/financing thwarts the dregs-skimming role of bailouts and undermines the overall effectiveness of bailouts. Not only do these firms withdraw from the second-period market, but their withdrawal also exacerbates the stigma for those participating in the second-period market. The consequence is devastating: *the second-period market freeze is even worse than if there were no bailout!* However, this does not necessarily mean that a bailout has no effect; it stimulates asset trade in the first period, and this effect may outweigh the dampening effect in the second period. Nevertheless, stimulation will be short-lived in this equilibrium. We show that the policy maker can avoid this equilibrium by offering sufficiently generous bailout terms—i.e., high purchase prices for assets, in which case the stigma will manifest itself in a different form: “delayed stimulation.”

A delayed stimulation equilibrium arises when high-quality firms refuse to sell either to the government or to the market in first period. They do so to build a good reputation on their assets so that they can sell their assets in the second period at favorable terms. This

⁴The withdrawal from the $t = 2$ funding market should not be literally interpreted as a firm exiting from the funding market altogether. Our model is (inevitably) stylized, so the results should be interpreted with some care. In practice, a firm’s withdrawal from the funding market will more realistically correspond to its cutting back on additional projects *at the margin* that it would otherwise pursue.

equilibrium is possible when the bailout offer is generous enough to attract a large fraction of firms with lower-quality assets. This allows firms to send a credible signal about their asset quality when they *refuse* the bailout offer. Buyers will then respond with a very attractive price offer in the second period—one that makes it worthwhile for high-quality firms to forego asset sale in the first period. In sum, the equilibrium endogenously creates an opportunity for high-quality firms to signal their financial strength by rejecting the government’s generous offer.

Such a favorable signaling opportunity offsets the adverse effect of bailout stigma, even though market rejuvenation is delayed to the second period. The presence of firms rejecting a bailout means that the volume of asset trade is lower in the first period than in the one-period benchmark with the same bailout offer. In an extreme case, it is even possible that the bailout has no stimulation effect in the first period relative to the *laissez-faire* economy. Such an initial lack of response may be seen as a policy failure. However, the policy is “quietly” strengthening the confidence (in the refusing firms) and bears dividend in the second period. In fact, the overall trade volume is higher than in a short-lived stimulation equilibrium; remarkably, it is the same *as if there were no bailout stigma*—that is, if the identities of bailout recipients were concealed successfully, which is often difficult to achieve in practice. Except for the delay, rejection of bailouts by high-quality firms could very well be a blessing in disguise.

The conclusion that the policy maker may wish to offer a generous bailout term to induce a delayed stimulation equilibrium is further reinforced once the costs of bailouts are taken into account. Even though the bailout term required for a short-lived stimulation equilibrium is more modest, the policy ends up being costly since it induces high-type firms which would not otherwise require a bailout to accept a bailout at terms that would compensate their stigma. For this reason, the delayed stimulation equilibrium, if it can be induced, achieves the same stimulation effect as short-lived stimulation equilibrium at a lower cost. Our theory thus recognizes the need for bailout terms to be sufficiently generous to yield a tangible benefit. This implication, although departing from the classical Bagehot’s rule,⁵ is consistent with the approach taken by the policy makers in the Great Recession.

The remainder of the paper is organized as follows. Section 2 presents our model while Section 3 analyses several benchmark cases. In Section 4, we study various equilibria under government intervention. Section 5 provides the welfare analysis of bailout policy. Section 6 discusses related literature. Section 7 concludes. Proofs not contained in the main text are deferred to Appendix A and Online appendix.

⁵Bagehot’s rule, originating from the 1873 book, *Lombard Street*, by William Bagehot, prescribes that central banks should charge a higher rate than the markets to discourage banks from borrowing once the crisis subsides. Bailout stigma was not a serious issue in 1873, however, since the regulatory system in 1873 Britain ensured concealment of the identities of emergency borrowers, as Gorton (2015) points out.

2 Model

We adopt a model that extends [Tirole \(2012\)](#) to a setup which admits a bailout stigma. There is a continuum of firms each endowed with two units of legacy assets with the same value; one unit of the asset becomes available in each of two periods ($t = 1, 2$) for possible sale.⁶ The asset value θ of each firm is privately known to that firm and distributed on $[0, 1]$ according to cdf F with density f . For convenience, we hereafter call a firm with legacy asset θ a type- θ firm. Throughout, we assume that f is log-concave, i.e., $d^2 \log f(\theta)/d\theta^2 < 0$. Log-concavity of f implies intuitive properties we shall use on truncated conditional expectation: for any $0 < a < b < 1$, $0 < \frac{\partial}{\partial a} \mathbb{E}[\theta|a \leq \theta \leq b]$, $\frac{\partial}{\partial b} \mathbb{E}[\theta|a \leq \theta \leq b] < 1$ (see [Bagnoli and Bergstrom \(2005\)](#)). We additionally assume that for each $b \in (0, 1]$, $2\mathbb{E}[\theta|a < \theta < b] - \mathbb{E}[\theta|\theta \leq a]$ is increasing in a for any $a \in (0, b)$. These properties, which hold for many well-known distributions, facilitate the characterization of our equilibria.

In each period, an investment project becomes available to each firm. The project is socially valuable with *net* return $S > 0$ but requires funding of $I > 0$. The firm can finance the project by selling its legacy asset each period. As we will see, the outcome from this *laissez-faire* regime will typically be inefficient due to the adverse selection associated with uncertain asset value. This inefficiency rationalizes a government bailout in the form of an offer to purchase legacy assets at some price p_g . The government purchase price p_g is initially exogenous (at level above I); we later discuss how it may be chosen optimally in [Section 5](#) in light of the public cost of a bailout. The timeline of our full game is depicted in [Figure 1](#):

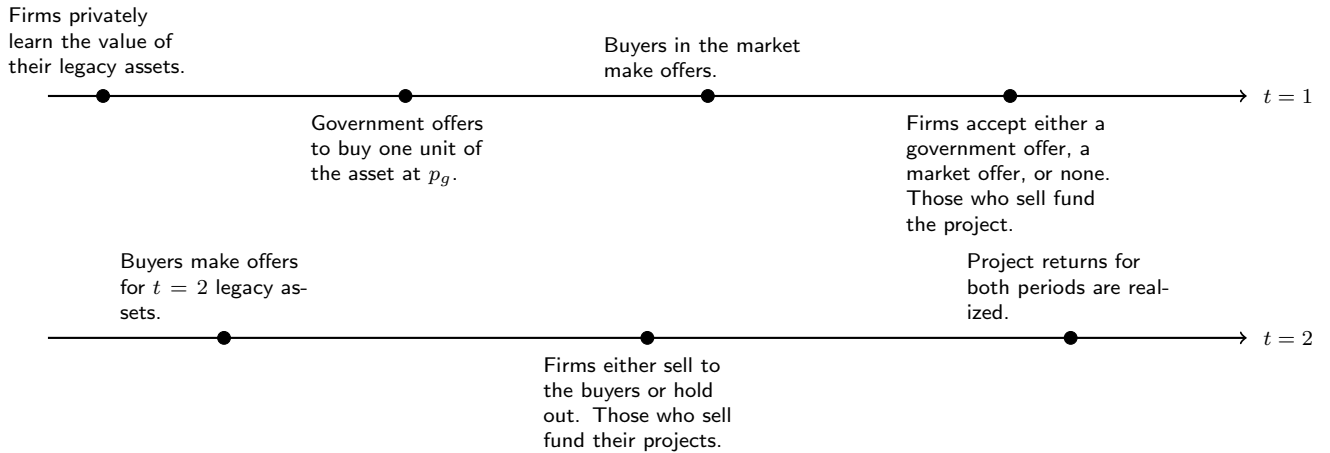


Figure 1 – Timeline for the two-period model

To focus our attention on the main issue—namely, bailout stigma—we make several simplifying assumptions.

⁶One can think of the assets as account receivable or the contract for (securitized) assets to be delivered over two periods.

First, just like [Tirole \(2012\)](#), we assume that the limited pledgeability of the project inhibits direct financing. This means that the sale of legacy assets is the only means of funding the project for firms. In the same vein, we consider the government’s purchase of legacy assets as the only means of government bailout. This is primarily a simplifying assumption. As shown by [Philippon and Skreta \(2012\)](#), the main thrust of the analysis extends to the case in which the project can be pledged along with legacy assets as collateral to obtain financing.⁷ From this broader perspective, adverse selection with respect to legacy assets must be interpreted as pertaining to their values as collaterals required for financing; and our results can be translated into this broader context naturally.

Second, project returns are realized at the end of $t = 2$, so it is impossible to use the return from $t = 1$ project to finance the $t = 2$ project. This assumption is made primarily to simplify the analysis but is well justified in many settings in which there are differences between accrual and realization of cash flows.⁸ That is, we consider the case where the project return accrues each period if it is funded but the final cash flow realizes in $t = 2$.

Third, as is standard in the literature, we assume that asset buyers are competitive and make purchase offers in Bertrand fashion. We assume that, in the event of an indifference, a buyer breaks a tie in favor of buying an asset rather than not buying.⁹ Buyers live for one period and make offers that would break even in expectation. Importantly, the buyers in $t = 2$ can make rational inference about firms’ types from their observable behavior in $t = 1$, in particular with their acceptance/rejection of a bailout offer.

Fourth, we assume that the sale of assets to the market in $t = 1$ is private and therefore not revealed to buyers in $t = 2$. This implies that buyers in the $t = 2$ market cannot distinguish between those that sold in the $t = 1$ market and those that did not. Again the primary reason for this assumption is to simplify the analysis by shutting off channels of dynamic information revelation. But this assumption is well justified given that many important financial and real assets are sold privately over the counter. The main thrust of our results extends to the case of observable sale, as shown by the working paper version of our paper (see [Che, Choe and Rhee \(2018\)](#).) Further, this assumption makes a comparison with [Tirole \(2012\)](#) transparent, thus helping to isolate the effect of stigma.

Fifth, a government bailout is available only in the first period. This is consistent with the observed practice: governments refrain from engaging in long-term bailouts and from

⁷Their insight appears to apply to our context, which suggests that debt contracts would be optimal in our context as well. Since the stigma issue is separable from the issue of contract form, we abstract from it in the current paper.

⁸If a project return from $t = 1$ can be used to fund the project in $t = 2$, then this will affect the amount of funding each firm demands based on the successful funding in the first period. This complicates analysis in a way that does not add any obvious new insight to our central theme. In particular, bailout stigma will remain relevant as long as the funding need is not fully met by the cash flow generated by an early project.

⁹This tie-breaking assumption is largely to simplify analysis and exposition. It has no material effect on the substantive results obtained in the paper.

complete “nationalization” of distressed firms (which would be equivalent to purchasing two units of the asset in our model). Further, our goal is to study the reputational consequence of accepting a bailout, which can be studied most effectively when no bailout is available in the second period.

Last, we focus on “transparent” bailouts; namely, the $t = 2$ market observes which firms have accepted the bailout. Not only do transparent bailouts highlight the stigma effect most clearly, but they are also important from a practical perspective. While secret bailouts may address the stigma problem, secrecy is often difficult to achieve in practice.

3 Preliminary Analysis

Before proceeding, we study several benchmarks. They will facilitate comparison with, and provide a context for, our main results, which will follow in Section 4.

3.1 *Laissez Faire* without Government Bailout

We first consider the benchmark without a bailout. The timeline is the same as Figure 1, except that the government’s bailout is absent. Since the sale of the assets is private and not publicly revealed, there is no linkage between the market outcomes across two periods. Thus, the game reduces to a one-period game (repeated twice) whose equilibrium coincides with that of Tirole’s game without bailout.

Fix any period. The equilibrium outcome is understood best as a form of Akerlof’s lemons problem, which is depicted in Figure 2. The figure plots two curves both as functions of the marginal type of firm $\hat{\theta}$ selling to the market. The marginal type $\hat{\theta}$ effectively represents the “quantity” sold since types θ sell if and only if $\theta \leq \hat{\theta}$ in equilibrium.¹⁰ The marginal type faces $\hat{\theta} - S$ as the opportunity cost of selling: by selling the firm loses the asset of value $\hat{\theta}$ but gains the net surplus S . Since the marginal type $\hat{\theta}$ must be indifferent to selling in equilibrium, we have $p = \hat{\theta} - S$, giving rise to the *supply curve*. Meanwhile, buyers of asset quality $\theta \leq \hat{\theta}$ enjoy benefit $\mathbb{E}[\theta | \theta \leq \hat{\theta}]$ on average. Bertrand competition among buyers means that average benefit must equal price in equilibrium, giving rise to the *average benefit curve*.

Clearly, supply and average benefit curves must intersect at the equilibrium marginal type $\hat{\theta} = \theta_0$, where θ_0 satisfies

$$\theta_0 - S = \mathbb{E}[\theta | \theta \leq \theta_0] =: p_0. \tag{1}$$

The log-concavity assumption means that the average benefit curve always crosses the supply

¹⁰This feature follows from the single-crossing property: if a type- θ firm sells, then type- $\theta' < \theta$ firm strictly prefers to sell. The quantity sold is thus $F(\hat{\theta})$ which corresponds to $\hat{\theta}$ in one-to-one manner.

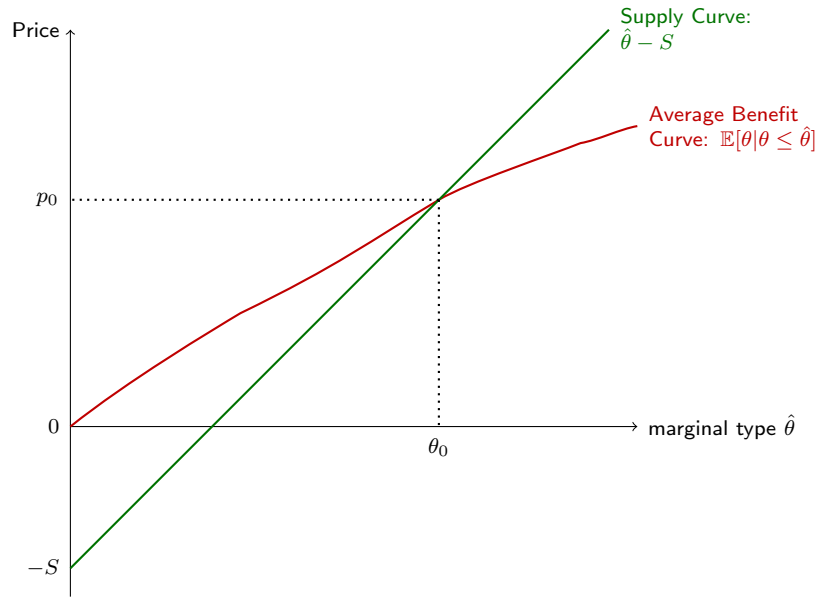


Figure 2 – Determination of the cutoff type θ_0 in the laissez-faire economy

curve from above. Hence, there is a unique threshold θ_0 satisfying this requirement¹¹—hence, a unique equilibrium. Finally, for trade to occur in equilibrium, price p_0 must be at least I , or else there are no gains from trade.

Figure 3 summarizes the equilibrium configuration.¹²

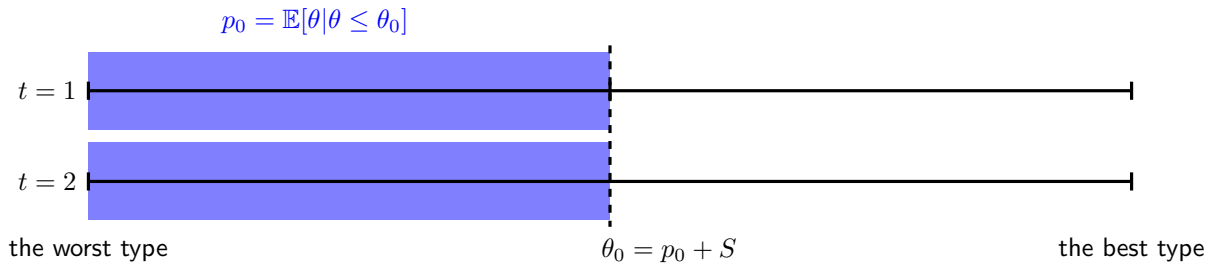


Figure 3 – No bailout equilibrium

Adverse selection means that the above outcome is typically inefficient. Specifically, if $S < 1 - \mathbb{E}[\theta]$, then $\theta_0 < 1$, so not all firms sell and finance their projects. It is also possible for $\theta_0 = 0$, in which case the market freezes completely. To focus on the nontrivial case, we

¹¹It is routine to check that if f is log-concave ($\frac{\partial^2 \log f(\theta)}{\partial \theta^2} < 0$ for all θ), then there is a unique θ satisfying (1); see [Tirole \(2012\)](#).

¹²As mentioned, buyers cannot update their information since the market transactions are private. If market transactions were observable, then trading decisions become dynamic, which makes analysis complicated; see [Che, Choe and Rhee \(2018\)](#).

assume $\theta_0 < 1$. For expositional ease, it is also convenient to focus on the partial freeze case ($\theta_0 > 0$) in what follows. We will discuss the full freeze case ($\theta_0 = 0$) later in Remark 2.

Example 1. Consider the uniform case, i.e., $F(\theta) = \theta$. In this case, the equilibrium price p_0 is determined by $p_0 = \mathbb{E}[\theta | \theta \leq \theta_0] = \frac{\theta_0}{2}$. Suppose $\theta_0 \in (0, 1)$. Then, from the indifference condition $\theta_0 = p_0 + S = \frac{\theta_0}{2} + S$, the cutoff type θ_0 is uniquely determined as $\theta_0 = 2S$, and $p_0 = S$, if $S \in [I, 1/2)$. If $S < I$, then the equilibrium price $p_0 = S$ cannot fund the project, so the market fully freezes, and hence $\theta_0 = 0$. If $S \geq 1 - \mathbb{E}[\theta] = 1/2$, then $\theta \leq \mathbb{E}[\theta] + S$ for all $\theta \in [0, 1]$, and therefore, $\theta_0 = 1$.

3.2 Bailout without Stigma: One-Period Model

We next consider another benchmark, the one-period bailout model by [Tirole \(2012\)](#) in which the government offers to purchase assets at price p_g above the laissez-faire price p_0 before the market opens. Specifically, the timeline simply comprises $t = 1$ in Figure 1.¹³ Since there is no consequence of accepting a bailout from the government in this one-period model, there is no bailout stigma—at least in the sense we will capture in our two-period model later.¹⁴ Thus, this benchmark will help to identify the role of bailout stigma later in our main analysis.

A Perfect Bayesian equilibrium, or called simply an equilibrium from now on, of this game is characterized as follows. Let μ_g and μ_m denote the fractions of types $\theta \leq p_g + S$ that sell to the government and to the market, respectively, where $\mu_g + \mu_m = 1$. We can first argue that $\mu_g > 0$. If no firm accepts the government offer, then the laissez-faire equilibrium will prevail, with marginal type θ_0 and equilibrium price $p_0 = \mathbb{E}[\theta | \theta < \theta_0]$. Since $p_g > p_0$, however, firms will deviate to accept the government offer, a contradiction.

Next, suppose $\mu_m > 0$, so the market is active in equilibrium.¹⁵ Then, the market price p_m must equal the government price p_g , or else a lower offer will not be accepted. Given this, firms must sell (either to the government or to the market) if and only if $\theta < p_g + S$. Let $\bar{\theta}_g$ and $\bar{\theta}_m$ denote the average values of assets sold to the government and the market, respectively

¹³An astute reader will notice that this timeline differs slightly from that considered by [Tirole \(2012\)](#), where the market opens after firms have decided on the government offer. We adopt the current timeline since it is arguably more realistic, and also it permits equilibrium existence more broadly for our two-period extension. For the one-period version, the difference is immaterial, since the equilibrium under the current timeline is payoff-equivalent to Tirole’s equilibrium for all players involved. In addition, we do not invoke an equilibrium refinement adopted in [Tirole \(2012\)](#), as the central feature of the equilibrium holds irrespective of the refinement. See Remark 1.

¹⁴Note [Tirole \(2012\)](#) and [Philippon and Skreta \(2012\)](#) do recognize “stigma” associated with the types of firms that accept a bailout, but they do not study its effect on the subsequent game as well as on the initial decision to accept the bailout, the dual focuses of the current paper.

¹⁵Under our timeline, the market may not be active in equilibrium. To see how such an equilibrium can be supported, suppose a buyer deviates and offers a price $p' > p_g$. Since firms have not yet accepted the government offer by then (given our timeline), all types $\theta < p'$ would accept the deviation offer, and the deviating buyer will suffer a loss since $p' > p_g > p_0$.

($\bar{\theta}_m$ can be arbitrary in case $\mu_m = 0$). Clearly, we must have

$$\mu_g \bar{\theta}_g + \mu_m \bar{\theta}_m = \mathbb{E}[\theta | \theta \leq p_g + S]. \quad (2)$$

Further, since the market buyers must break even in case $\mu_m > 0$, we have $p_m = \bar{\theta}_m$. Given $p_g = p_m$, this in turn implies $\bar{\theta}_m = p_g$.

The central feature of the [Tirole \(2012\)](#) follows from these observations:

Theorem 1 (dregs-skimming).

- (i) *In any equilibrium of the one-period benchmark with government offer $p_g > \max\{p_0, I\}$, firms sell assets (either to the government or to the market) at price p_g if and only if $\theta < p_g + S$. Since $p_g > p_0$, more firms finance their projects than without the government intervention.*
- (ii) *If $\mu_m > 0$ so that the market is active, then we must have $\bar{\theta}_g < \bar{\theta}_m$; i.e., on average lower value assets are sold to the government than to the market.*

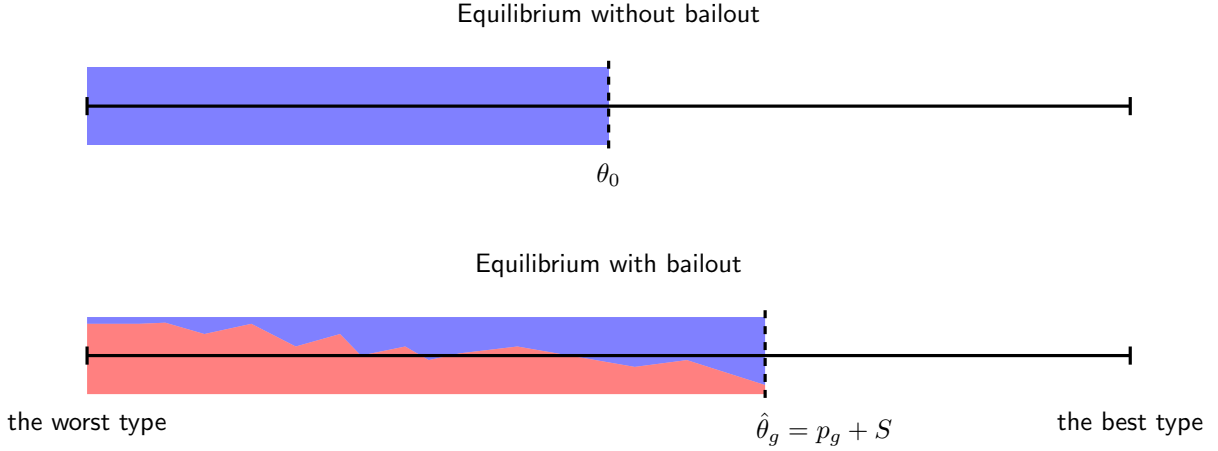
Proof. We have already established (i). To prove (ii), suppose to the contrary $\bar{\theta}_m \leq \bar{\theta}_g$. Then, by (2), $\bar{\theta}_m \leq \mathbb{E}[\theta | \theta \leq p_g + S]$. Since $\bar{\theta}_m = p_g$, we have $p_g \leq \mathbb{E}[\theta | \theta \leq p_g + S]$, or $p_g + S \leq \mathbb{E}[\theta | \theta \leq p_g + S] + S$. By the definition of θ_0 , $p_g + S \leq \theta_0 = p_0 + S$, which contradicts $p_g > p_0$. The remaining characterizations follow from the observations preceding the theorem. *Q.E.D.*

Figure 4 illustrates the outcomes with and without government bailout. By offering a higher price p_g than the laissez-faire price p_0 , the government does indeed take out relatively low-value assets, which in turn improves the perception of the assets sold to the market and thus alleviates adverse selection.

More importantly for our purpose, assets are sold to the government at the same price as they are sold to the market. This reflects the absence of stigma associated with accepting a bailout. Plainly, in the one-period problem, firms that accept the bailout do not have any consequences to worry about simply because the game ends after the bailout.

Remark 1 (The role of the equilibrium refinement in [Tirole \(2012\)](#)). *To obtain the “dregs-skimming” role of bailout, [Tirole \(2012\)](#) invokes an equilibrium refinement—that the market sale collapses with an arbitrarily small probability. This refinement “forces” the equilibrium to have the dregs-skimming feature, since it implies single-crossing: namely, there exists $\tilde{\theta} \in (0, p_g + S)$ such that types $\theta \leq \tilde{\theta}$ all sell to the government and types $\theta \in (\tilde{\theta}, p_g + S]$ all sell to the market.¹⁶ However, this refinement obfuscates the source of dregs-skimming: namely,*

¹⁶Suppose a market sale is subject to probability $\epsilon > 0$ of cancellation. If a type- θ firm prefers to sell to the government, then $p_g + S \geq (1 - \epsilon)(p_m + S) + \epsilon\theta$, where p_m is the equilibrium market price. This means that all types $\theta' < \theta$ must strictly prefer to sell to the government.



Note: the types selling to a market is depicted by blue and the types selling to the government is depicted by red.

Figure 4 – Effects of bailout in the one-period benchmark

whether it is an artifact of the refinement or something more fundamental. Without invoking that refinement, Theorem 1 proves that dregs-skimming is fundamental (and not driven by the refinement). Without the refinement, however, there are multiple equilibria that differ in terms of μ_g and the value $\bar{\theta}_g$, but every such equilibrium exhibits the “dregs-skimming” feature.

3.3 Secret Bailouts

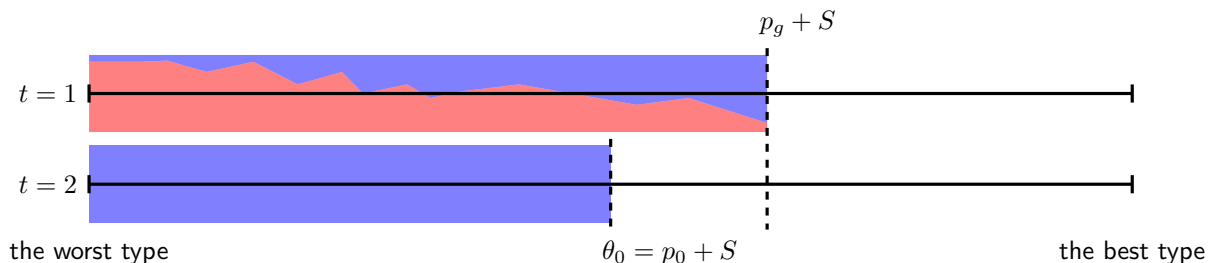
In order to identify the effects of bailout stigma, we need to understand what happens if the policy maker can eliminate the stigma altogether. Imagine that the policy maker “completely and successfully” conceals the identities of the firms that accept the government offer. This kind of secrecy has been an important part of the bailout policy, precisely because of the stigma issue.¹⁷ In this sense, secret bailouts are worth studying in their own right. Nevertheless, we primarily regard secrecy as a benchmark against which transparent bailouts are compared, given our premise that “complete” secrecy has been so far difficult to achieve despite many

¹⁷Gorton and Ordoñez (forth) supports such a policy. During crises, debt contracts lose “information insensitivity” as investors scrutinize the downside risk of underlying collaterals, leading to an adverse selection. They argue that withholding information on whether borrowers borrow from discount windows of central banks can make debtors less information sensitive and alleviate adverse selection. As will be seen, secrecy has a more nuanced effect in our model.

concerted efforts.¹⁸

The equilibrium under complete secrecy is very easy to analyze. Since neither selling to the market nor selling to the government is observed, the former by assumption and the latter by secrecy, firms need not worry about the signaling consequences of their $t = 1$ actions. Hence, the equilibrium in $t = 1$ coincides with that of Theorem 1. Given no informational leakage, the outcome of $t = 2$ coincides with the no-intervention benchmark.

Theorem 2 (Secret bailouts). *Suppose the government offers to purchase assets at $p_g > p_0$ with full secrecy. Then, in equilibrium, firms accept the government offer in $t = 1$ if and only if $\theta < p_g + S$. In $t = 2$, firms sell assets to the market at price p_0 if and only if $\theta < \theta_0$.*



Note: the types selling to a market is depicted by blue and the types selling to the government is depicted by red.

Figure 5 – Equilibrium with secret bailouts

4 Government Bailout and Stigma

We now turn to the two-period game whose timeline is depicted in Figure 1. We continue to assume that the government offer is above the laissez-faire price: $p_g > p_0$. Otherwise, there is only a trivial equilibrium in which the laissez-faire outcome prevails, with no firms accepting the government offer.

¹⁸The identities of banks borrowing from the discount window facilities (DW) are occasionally leaked to either the news media or the market participants through a number of channels. First, despite the apparent secrecy attached to DW, the access to DW by borrowing firms has been detected by news media (Armantier et al., 2015; Berry, 2012). For instance, the Financial Times reported the news that Deutsche Bank had borrowed from DW one day ago (see “Fed fails to calm money markets,” *The Financial Times*, August 20, 2007). Second, the market participants can identify DW borrowers from these banks’ market activities or the information released by the Fed. On its weekly report, the Fed discloses whether there is an increase in aggregate DW borrowing. In addition, financial institutions can observe whether a bank did not borrow or lend at the federal funds market at that time. Combining all the information, one can easily identify a DW borrower (Haltom, 2011).

We begin by analyzing the structure of a possible equilibrium. We focus on the equilibrium that is obtained in the limit as the relative weight $\delta < 1$ for the $t = 2$ payoff approaches 1.¹⁹

Lemma 1. *In any equilibrium with $p_g > p_0$, there are three cutoffs $0 < \hat{\theta} \leq \hat{\theta}_g \leq \theta_2$ such that types $\theta \leq \hat{\theta}$ sell assets in both periods, some measure of whom sell to the government and the remaining measure sell to the market in $t = 1$; types $\theta \in (\hat{\theta}, \hat{\theta}_g]$ sell only to the government in $t = 1$ but do not sell in $t = 2$; types $\theta \in (\hat{\theta}_g, \theta_2]$ sell only in $t = 2$; and types $\theta > \theta_2$ never sell their assets in either period.*

The structure of an equilibrium is depicted in Figure 6.

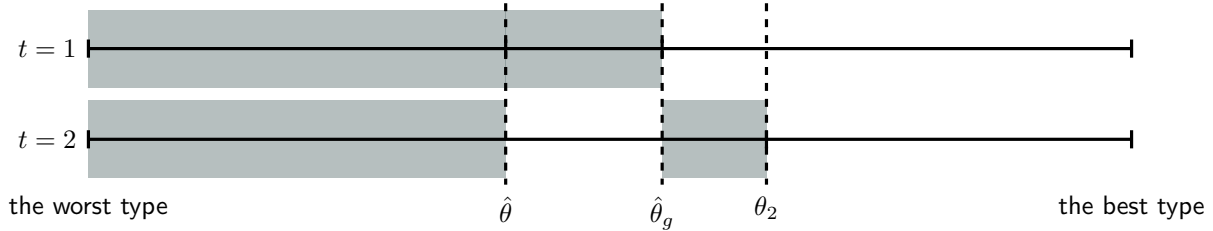


Figure 6 – General structure of equilibrium

Lemma 1 rests on several observations. First, firms’ preferences satisfy the single-crossing property, implying that a lower type has more incentives to sell than a higher type in either period; so the total number of units sold in equilibrium across the two periods must be non-increasing in θ . Second, the fact that buyers (either the government or the market) never ration sellers means that the quantity traded for each firm must be either zero or one in each period. Third, an arbitrarily small discounting of the second-period payoff, along with the first two observations, implies that, among those that sell only in one period, early sellers are of lower types than late sellers. These observations give rise to the stated cutoff structure, as depicted in Figure 6. We omit the formal proof since it follows from a standard argument based on these observations.

In what follows, we limit attention to the case where p_g is not so high—more precisely $p_g < 1 - S$ —that all firms accept the bailout.²⁰ Then, Lemma 1 implies that there are only two

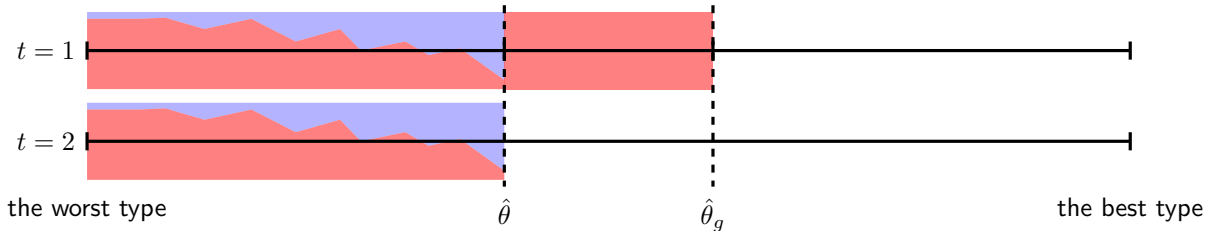
¹⁹The assumption of $\delta < 1$ is meant to capture the fact that even though the reputational consequence of accepting a bailout may be important, its effect does not outweigh the direct payoff consequence of the decision, which appears to be first-order. Further, the reputational impact attached to bailouts does not usually persist after the financial crises. For instance, the total TARP bank funds outstanding, after the launch of TARP in October 2008, reached peak at \$235.3 billion in February 2009, but sharply decreased to \$80.4 billion in January 2010 (go to <https://www.treasury.gov/initiatives/financial-stability/reports/Pages/TARP-Tracker.aspx> for more details). This observation indicates that the banks, after having joined TARP during 2007-2009 Great Recession, had little trouble in funding at the markets after the crisis was over.

²⁰Given the condition, we will have $\hat{\theta}_g < 1$. In case p_g is higher so that all firms accept the bailout, the second-period would coincide with the laissez-faire outcome. Such a boundary case is unrealistic in addition to being very costly from a welfare perspective, as we discuss later.

possible types of equilibria: (a) *short-lived stimulation* equilibria and (b) *delayed stimulation* equilibria, depending on whether the stimulation effect of a bailout arises in $t = 1$ or delayed to $t = 2$. More formally, these two types of equilibria correspond to the cases of $\hat{\theta}_g = \theta_2$ and of $\hat{\theta}_g < \theta_2$, respectively, in the cutoff structure characterized in Lemma 1. In the Online Appendix B, we formally show that these are the only possible types of equilibria.

4.1 Short-lived Stimulation Equilibria

This type of equilibrium corresponds to the case of $\hat{\theta}_g = \theta_2$ in Lemma 1, and is depicted in Figure 7. Importantly, the segment $[\hat{\theta}_g, \theta_2]$ of firms selling in $t = 2$ in Lemma 1 (i.e., delayed selling) is missing in this equilibrium. Consequently, a bailout triggers an immediate increase in trade volume in $t = 1$, but, as we will argue, its effect will be short-lived.



Note: the types selling to a market is depicted by blue and the types selling to the government is depicted by red.

Figure 7 – Short-lived stimulation equilibrium

For the purpose of characterization, we suppose that an equilibrium of this type exists, and investigate its properties. Specifically, fix an equilibrium in which the cutoffs defined in Lemma 1 satisfy $0 < \hat{\theta} \leq \hat{\theta}_g = \theta_2 < 1$, as illustrated in Figure 7. First, consider types $\theta \leq \hat{\theta}$. Let μ_g and μ_m be the fractions of these firms selling to the government and to the market in $t = 1$, respectively, where $\mu_g + \mu_m = 1$. One can show that both fractions are strictly positive in equilibrium.²¹ Let $\bar{\theta}_g$ and $\bar{\theta}_m$ denote respectively the mean values of the assets sold by the two groups. First, $p_2^g = \bar{\theta}_g \geq I$, or else these firms would not sell in $t = 2$, a contradiction to

²¹First, suppose $\mu_m = 0$. Then, all types $\theta \leq \hat{\theta}_g$ sell to the government in $t = 1$. But then, the holdouts in $t = 1$ would be revealed in $t = 2$ to have types $\theta > \hat{\theta}_g$. Given $\hat{\theta}_g < 1$, a positive measure of them will attract buyers offering price higher than $\hat{\theta}_g$. This leads to $\theta_2 > \hat{\theta}_g$, a contradiction to the type of equilibrium we are considering. Next, suppose $\mu_g = 0$, hence all types $\theta \leq \hat{\theta}$ sell to the market. We cannot have $\hat{\theta} = \hat{\theta}_g$, since then no firm accepts bailout, and the laissez faire cannot support such a cutoff $\hat{\theta}_g \geq p_g + S$, where the inequality is obtained earlier, since $p_g + S > p_0 + S = \theta_0$. Hence, $\hat{\theta} < \hat{\theta}_g$. But then, bailout recipients are revealed to have types $\theta \geq \hat{\theta}$, hence attract offers higher than $\hat{\theta}$ in $t = 2$, which contradicts the definition of $\hat{\theta}$ in Lemma 1.

Lemma 1. Also, by definition,

$$\mu_g \bar{\theta}_g + \mu_m \bar{\theta}_m = \mathbb{E}[\theta | \theta \leq \hat{\theta}]. \quad (3)$$

Next, let p_m denote the price firms receive from selling to the market in $t = 1$. The $t = 2$ market price depends on whether or not a firm received the bailout in $t = 1$, as these events are observed by buyers in $t = 2$. Let p_2^g and p_2^m denote respectively the prices offered in $t = 2$ to those that accepted the bailout and to those that did not in $t = 1$. In the short-lived stimulation equilibrium, no firms sell only in $t = 2$, so the latter group consists of only those that sold to the market in $t = 1$. Since buyers break even in expectation, we must have $p_2^g = \bar{\theta}_g$. Similarly, $p_m = p_2^m = \bar{\theta}_m$, since those that sold to the market in $t = 1$ are also believed correctly to of type $\bar{\theta}_m$ on average in both periods. Further, those firms selling in both periods must be indifferent between accepting the bailout and selling to the market in $t = 1$:

$$p_g + p_2^g + 2S = p_m + p_2^m + 2S \iff p_g + \bar{\theta}_g = 2\bar{\theta}_m. \quad (4)$$

Next, suppose further that $\hat{\theta} < \hat{\theta}_g$. Then, the cutoff type $\hat{\theta}$ must be indifferent between selling to the market in both periods and accepting the bailout in $t = 1$ and not selling in $t = 2$:

$$2\bar{\theta}_m + 2S = \hat{\theta} + p_g + S. \quad (5)$$

Lastly, either the cutoff $\hat{\theta}_g$ must be one or else the type $\hat{\theta}_g$ must be indifferent between accepting the bailout in $t = 1$ and not selling in $t = 2$ and not selling in either period. Hence,

$$\hat{\theta}_g = (p_g + S) \wedge 1. \quad (6)$$

From the necessary conditions on the cutoff types above, we can derive the following properties of short-lived stimulation equilibria.

Theorem 3 (*Short-lived stimulation equilibria*). *Suppose there exists a short-lived stimulation equilibrium given $p_g > p_0$. Then,*

- (i) $\bar{\theta}_g < \bar{\theta}_m < p_g$;
- (ii) $\hat{\theta} < \theta_0 < \hat{\theta}_g = p_g + S$;
- (iii) $p_g < 2\theta_0 - \mathbb{E}[\theta | \theta \leq \theta_0]$.

Proof. See Appendix A.

Q.E.D.

We highlight three features of short-lived stimulation equilibria. First, bailout recipients suffer stigma. In the $t = 2$ market, bailout recipients are believed to be of type $\bar{\theta}_g$ (on average), while those that sell to the market in $t = 1$ are believed to be of type $\bar{\theta}_m > \bar{\theta}_g$. Thus, assets held by bailout recipients are sold at discount precisely equal to $\Delta = \bar{\theta}_m - \bar{\theta}_g$, since the market correctly infers the difference in their average asset values. Of course, this stigma must be compensated in $t = 1$, or else no firms would accept the bailout. In particular, the government must pay more than the market does for the asset in $t = 1$. Since the government offer is fixed at p_g , this means that the market in $t = 1$ must clear at price $p_g - \Delta$. In other words, buyers demand a haircut Δ from firms selling to them for avoiding that stigma. Since the market is competitive, buyers cannot earn positive profit, so what this simply means is that the average type $\bar{\theta}_m$ of firms selling to the market must (endogenously) equal $p_g - \Delta$. It thus follows that the bailout premium must precisely compensate the stigma, as is stated in (4).

Second, dregs-skimming by government bailout—featured prominently in Tirole’s model—does not occur here. Bailout stigma here creates the incentive for high-type firms to mitigate it or avoid it altogether. Selling to the market in $t = 1$ instead is one option, but it is subject to a mark-down of asset price by Δ ; effectively, bailout stigma has “spread” to market sellers in $t = 1$. Another way to avoid the collateral damage is to accept the bailout but simply withdraw from the $t = 2$ market. Indeed, types $(\hat{\theta}, p_g + S]$ find it strictly profitable to accept the bailout but refuse to sell assets in $t = 2$ to avoid the stigma. Presence of these firms undercuts the government’s role to take out the most toxic assets and to boost the market reputation of the remaining firms. This has a long term effect, as we now turn to.

Third, as will be seen more clearly, the government bailout worsens the adverse selection in the $t = 2$ market relative to no bailout. The government’s inability to dregs-skim—or to take out the worst assets—makes stimulation short lived. In particular, high-type withdrawal from the $t = 2$ market to avoid stigma exacerbates the reputation of firms that *do* participate in the $t = 2$ market; they are effectively revealed to be of type $\bar{\theta}_g$ on average. In fact, this negative effect is so severe that the $t = 2$ market freezes more than if there were no bailout in $t = 1$: only types $\theta \leq \hat{\theta}$ sell in $t = 2$, where importantly $\hat{\theta} < \theta_0$. By comparison, all types $\theta \leq \theta_0$ would have traded in $t = 2$ in the absence of bailout (recall Figure 3). Clearly, transparency entails a strict loss of trade. Compared with a secret bailout, the volume of trade (and investment) induced under the transparent bailout is the same in $t = 1$ but strictly smaller in $t = 2$.

The properties identified so far are necessary but not sufficient for short-lived stimulation equilibrium. For a short-lived stimulation equilibrium to exist, additional conditions must be met. Specifically, buyers targeting bailout recipients should not gain from raising their offers to attract the boycotters (i.e., types $\theta \in [\hat{\theta}, p_g + S]$), and the buyers targeting non-recipients should have no incentives to raise their offers to attract holdouts (i.e., types $\theta > p_g + S$) together with the market sellers. These conditions are formally stated and shown to be

sufficient in Online Appendix C.1.

These conditions are not easy to check, so it is difficult to establish the existence of the equilibrium (or its sufficient condition) in a simple manner. Nevertheless, short-lived stimulation equilibrium exists for a range of p_g 's, for many common distribution functions F . For example, Figure 9-(a) in Section 4.3) shows (a continuum of) short-lived stimulation equilibria when F is uniform.

On the other hand, a short-lived stimulation equilibrium does not exist if p_g is sufficiently high. More precisely, as stated in (iii) of Theorem 3, the short-lived stimulation equilibrium disappears if $p_g \geq 2\theta_0 - \mathbb{E}[\theta|\theta \leq \theta_0]$. Roughly speaking, if p_g is sufficiently high, accepting the government offer becomes very attractive and this breaks the no arbitrage condition (4).²² One implication is that the policy maker can avoid triggering undesirable short-lived stimulation equilibria if she were to make the bailout offer sufficiently generous—a point that will become clear as we now turn to delayed stimulation equilibria.

4.2 Delayed Stimulation Equilibria

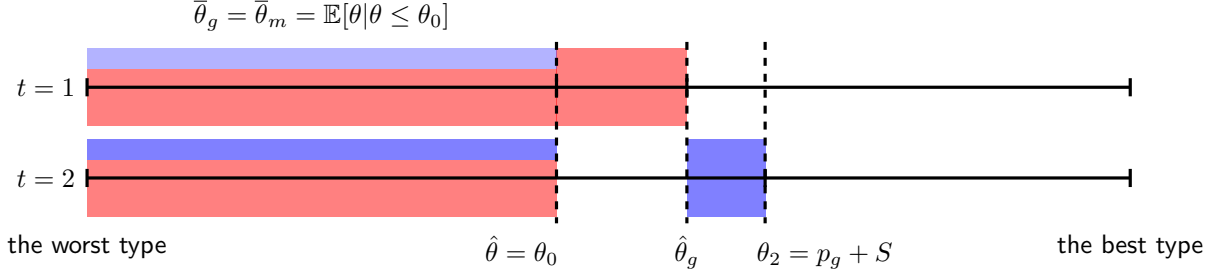
A delayed stimulation equilibrium has the structure that $\hat{\theta} \leq \hat{\theta}_g < \theta_2$ in the characterization in Lemma 1, and is illustrated in Figure 8. We call this *delayed stimulation* equilibrium since much of the stimulation effect materializes in $t = 2$. In particular, the highest type that trades does so in $t = 2$. Types $\theta < \hat{\theta}_g$ act similarly as in the short-lived stimulation equilibria: nonnegative fractions μ_g and μ_m of types $\theta \leq \hat{\theta}$ sell respectively to the government and to the market, and types $\theta \in (\hat{\theta}, \hat{\theta}_g)$ sell only to the government, where $\hat{\theta} \leq \hat{\theta}_g$.

What makes this equilibrium possible is the incentive that $t = 2$ buyers have to offer a sufficiently high price to attract high-type firms who hold out in $t = 1$. Such an incentive was absent in short-lived stimulation equilibria due to a sizable fraction μ_m of low-type market sellers. These firms cannot be distinguished from high-type hold-out firms and, therefore, would inflict a loss to buyers if they were to raise offers to attract high-type hold-out firms. In a delayed stimulation equilibrium, the fraction μ_m of low-type market sellers is sufficiently small, especially when p_g is large, so that $t = 2$ buyers do have an incentive to attract high-type holdouts, unlike in short-lived stimulation equilibria.

We now provide characterization of delayed stimulation equilibria.

Theorem 4 (*Delayed stimulation equilibria*). *In any delayed stimulation equilibrium, $\hat{\theta} = \theta_0$,*

²²To satisfy that condition—or equivalently, to support the sale at $t = 1$ market—the stigma $\Delta = \bar{\theta}_m - \bar{\theta}_g$ must increase, which further requires that firms selling to the $t = 1$ market must be of types close to $\hat{\theta}$. This in turn reduces the fraction μ_m of these firms. If this fraction shrinks sufficiently, it is no longer incentive compatible for buyers to buy only from these firms; it becomes profitable for buyers to deviate by raising their offers to attract high-type firms that held out in $t = 1$ (i.e., types $\theta > p_g + S$), thus breaking the short-lived stimulation equilibrium. See the proof of Theorem 3-(iii) in Appendix A for the supplementary analysis.



Note: the types selling to a market is depicted by blue and the types selling to the government is depicted by red.

Figure 8 – Delayed stimulation equilibrium

$\hat{\theta}_g \in [\hat{\theta}, p_g + S)$ and $\theta_2 = p_g + S$. In particular, the following holds.

- (i) Types $\theta \leq \theta_0$ sell in both periods, a positive (possibly the entire) measure of which accept the bailout.
- (ii) Among types $\theta \leq \theta_0$, those that sell to the market in $t = 1$ (if they exist) receive price p_0 in $t = 1$ and p_g from the $t = 2$ market, and those that accept the bailout sell assets at price p_0 in $t = 2$. Furthermore, these two groups of firms have the same average value of $\mathbb{E}[\theta | \theta \leq \theta_0] = p_0$.
- (iii) Types $\theta \in (\theta_0, \hat{\theta}_g]$ sell only in $t = 1$ and to the government at p_g .
- (iv) Types $\theta \in (\hat{\theta}_g, p_g + S]$ sell only in $t = 2$ at price p_g . Higher-type firms never sell in either period.

Proof. See Appendix A.

Q.E.D.

We discuss below several features of delayed stimulation equilibria. First, just as in short-lived stimulation equilibria, firms suffer from accepting a bailout. In $t = 2$, the market offers price p_0 to bailout recipients but $p_g > p_0$ to those that did not accept a bailout. But, unlike in short-lived stimulation equilibria, this differential treatment is not attributed to the difference in the average types between bailout recipients and market sellers in $t = 1$. Theorem 4-(ii) shows that the average asset value is precisely the same for these two groups of firms and equals $p_0 = \mathbb{E}[\theta | \theta \leq \theta_0]$. The differential treatment is instead due to the high-type holdouts in $t = 1$ that participate in the $t = 2$ market. Since buyers in $t = 2$ cannot distinguish these firms from the $t = 1$ market sellers, the latter firms receive a better offer.

Just as before, the differential treatment by the $t = 2$ market of bailout recipients vis-à-vis market sellers in $t = 1$ can sustain in equilibrium only if it is counterbalanced by the opposite treatment of these two groups in $t = 1$. Indeed, the government must pay more for the asset

than the market in $t = 1$ to compensate for the (relative) loss bailout recipients will suffer in $t = 2$. As before, the payoffs of those who sell in both periods are equalized. In short-lived stimulation equilibria, this payoff equalization, or no arbitrage, meant a “contagion of stigma” to all firms selling in both periods, which resulted in the worsening of the adverse selection in $t = 2$ than in the absence of bailout. This does not occur in delayed stimulation equilibria. It is because adverse selection is ameliorated in delayed stimulation equilibria due to high-type holdouts selling only in $t = 2$. As a result, bailout recipients are offered $p_0 = \mathbb{E}[\theta | \theta \leq \theta_0]$ in $t = 2$, exactly the same as the market offer that would prevail absent any bailout by the government.

The above discussions lead to the following key observation.

Corollary 1. *Each delayed stimulation equilibrium is equivalent to an equilibrium under secret bailout (described in Theorem 2) in total volume of asset sales—and thus in total investments undertaken by firms. The total volume of asset sales in any delayed stimulation equilibrium exceeds that in short-lived stimulation equilibria.*

Proof. By Theorem 4, the total volume of asset sales is $F(\theta_0) + F(p_g + S)$ in any delayed stimulation equilibrium, which equals that under a secret bailout. It also exceeds the total volume of asset sales in a short-lived stimulation equilibrium, $F(\hat{\theta}) + F(p_g + S)$, since $\hat{\theta} < \theta_0$. Q.E.D.

One interesting, and perhaps surprising, implication of this result is that delays in the effect of bailout should not be viewed as a policy failure, at least if one takes the secret bailout equilibrium as an ideal benchmark. Take a possible equilibrium where $\hat{\theta}_g$ is very low; in fact, one can show that $\hat{\theta}_g = \theta_0$ can be supported if p_g is not too high (again such an equilibrium arises due to a large mass of high type firms holding out in $t = 1$). In such an equilibrium, it may appear from the perspective of $t = 1$ commentators that bailouts have no impact, since trading and investment activity have not changed after the government offer. Their impression would be that the government purchase “crowded out” private purchase; a positive measure of firms with $\theta \leq \theta_0$ sell to the government at a higher price p_g than p_0 , the price they would have sold at in the absence of bailouts. Indeed, in the wake of the Great Recession, such a sentiment prevailed following the apparent lack of response by banks to the first wave of stimulation policies.²³

However, our result suggests that holdout by high-type firms (instead of taking the bailout and boycotting the $t = 2$ market) is a blessing in disguise. The presence of these holdout

²³In fact, as pointed out by Bolton, Santos and Scheinkman (2009), LIBOR-OIS spreads did not decrease following the implementation of the Fed’s emergency lending programs (such as the PDCF and the TSLF) during the 2007 – 2009 Great Recession. Bolton, Santos and Scheinkman (2011) also argued that the public liquidity programs, if implemented at a bad timing, will only crowd out the private liquidity supplied from the financial market. As can be seen, our theory provides a different perspective on the same phenomenon.

firms is precisely what leads the market to make very attractive offers to those that did not accept the bailout. Indistinguishable from these holdout firms, those that actually sold to the market in $t = 1$ also receive very attractive offers, thus overcoming their adverse selection. The alleviation of adverse selection for these firms in turn creates “collateral benefits” to those that accept the bailout, since no arbitrage means that they too can overcome stigma. Consequently, the increase in trade volume in $t = 2$ more than makes up for the initial lack of response, in comparison with short-lived stimulation equilibria.

From the policy maker’s perspective, bailouts could very well be seen as “working” mysteriously here. Paradoxically, the policy works here because it allows firms to send a strong signal on their financial strength by “rejecting” the bailout offer. This indirect signaling creates the desired stimulation effect later. As Corollary 1 suggests, the opportunity to “reject” a bailout turns out to be more effective than “accepting” one.

For a delayed stimulation equilibrium to exist, a buyer in each period should not have an incentive to deviate from the prescribed equilibrium strategies on every equilibrium path. Just as with short-lived stimulation equilibria, it is difficult to check these “no-deviation-by-buyers” conditions.²⁴ However, the equilibrium exists for a range of p_g ’s under general distribution functions.²⁵ For example, the equilibrium exists under the uniform distribution, as we show in Section 4.3.

Corollary 1 also shows that, if a p_g leads to a delayed stimulation equilibrium under the transparent bailout, it also yields the same total volume of trade as under a secret bailout. From this, one may conclude that bailout stigma need not be worrisome. However, there are two caveats to this conclusion. First, there is a possibility of multiple equilibria as we show in the next section. That is, the same p_g may also support a short-lived stimulation equilibrium, which is clearly undesirable as discussed previously. In order to avoid the selection of such an equilibrium, a policy maker may have to raise p_g beyond what she would otherwise offer. Second, a delay in and of itself may be undesirable for reasons not modeled in our theory. For instance, a prompt revitalization of economic activities often have external benefits for the rest of the economy. In particular, if we consider financial institutions investing in the real economy, a prompt restoration of their activities will have a positive spillover effect, and from this perspective delayed stimulation can be harmful. For these reasons, one may view the delay itself as a cost of stigma.

While it is difficult to find direct evidence on our equilibria, their main features are consistent with what transpired in the aftermath of the 2007–2009 Great Recession. For instance, consistent with our delayed stimulation equilibrium, firms outright rejected rescue offers made by the government ostensibly to signal their financial strength, as mentioned in the introduc-

²⁴Online Appendix C.2 presents these necessary conditions and show them to be sufficient for equilibrium.

²⁵We also show in Online Appendix C.2 that there always exists a delayed stimulation equilibrium if p_g is sufficiently high.

tion. In particular, our theory suggests that firms with high-quality assets are more likely to hold out when bailout terms are more generous (recall the difference between short-lived and delayed stimulation equilibria.) Indeed, there is some evidence that firms with high-quality assets participated less in programs that were more generous. [Krishnamurthy, Nagel and Orlov \(2014\)](#) find that during the Great Recession dealer banks with a large share of agency collaterals (i.e., collaterals guaranteed by the US government) rarely participated in the Primary Dealer Credit Facility (PDCF) despite their favorable funding rates, although they did participate in the Term Securities Lending Facility (TSLF) along with banks with a large share of non-agency collaterals (i.e., collaterals not guaranteed by the US government).

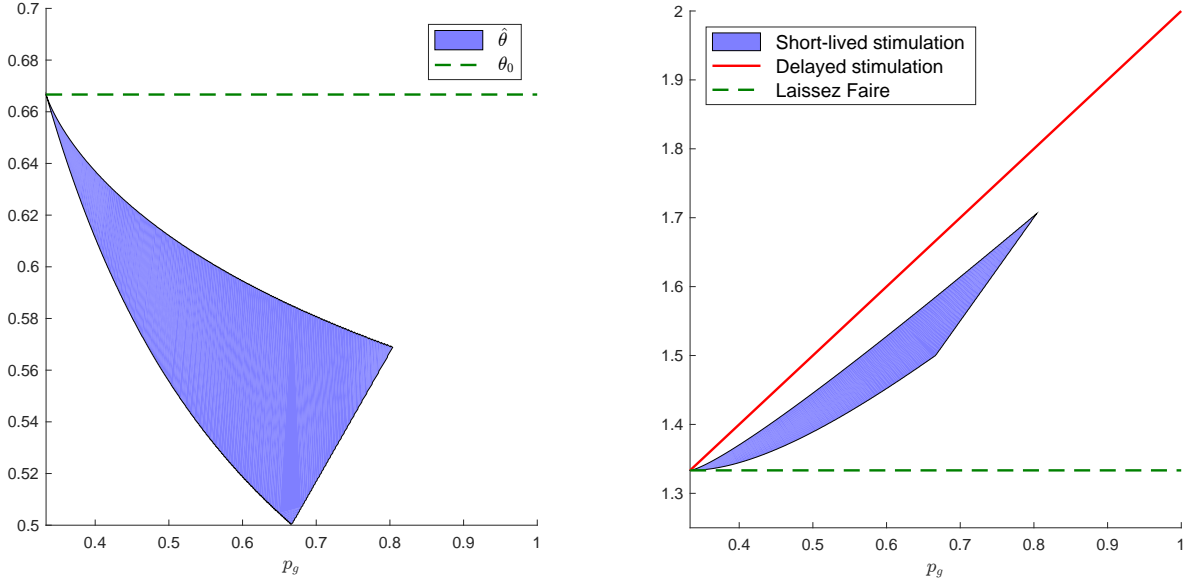
Remark 2 (The “full” freeze case: $\theta_0 = 0$). *We have so far implicitly assumed $\theta_0 > 0$, for ease of exposition. But our equilibrium characterizations from Theorems 3 and 4 extend even to the case of $\theta_0 = 0$; i.e., when the market would freeze completely absent any bailout. In this case, our characterizations imply that $\hat{\theta} = 0$, and this leads to existence of a unique equilibrium. Given $p_g \geq I$, the equilibrium admits a threshold $\hat{\theta}_g \in (0, p_g + S \wedge 1)$ such that types $\theta \in [0, \hat{\theta}_g]$ sell to the government in $t = 1$ but do not sell in $t = 2$, and types $\theta \in (\hat{\theta}_g, p_g + S \wedge 1]$ hold out in $t = 1$ but sell in $t = 2$ at price p_g , where $\hat{\theta}_g$ satisfies $\mathbb{E}[\theta | \hat{\theta}_g \leq \theta \leq p_g + S] = p_g$. Types $\theta > p_g + S \wedge 1$ sell in neither period. One can think of this as a form of delayed stimulation equilibrium: a bailout does not revive market in $t = 1$ but induces delayed trading in $t = 2$. In keeping with Corollary 1, the equilibrium induces the same trade volume as a secret bailout would.*

4.3 Uniform Distribution Example

To gain better understanding about the two types of equilibria and their possible coexistence, it is useful to exhibit them in full detail for some concrete parameter values. Assume uniform distribution $F(\theta) = \theta$, along with $S = 1/3$ and $I = 1/10$.

The left panel (a) of Figure 9 depicts the set of threshold types $\hat{\theta}$ supported in short-lived stimulation equilibria for various bailout terms p_g . The corresponding threshold for delayed equilibria is θ_0 , depicted as dotted line. The right panel (b) of Figure 9 depicts the total volume of trade induced by short-lived stimulation equilibria (blue area) and delayed stimulation equilibria (red line) for differing levels of p_g .

There are several interesting observations. First, as can be seen clearly by the blue area, there is a continuum of short-lived stimulation equilibria that induce different threshold values $\hat{\theta}$. Note also that short-lived stimulation equilibria exist for $p_g > p_0 = S = 1/3$ but not when p_g is sufficiently high. Specifically short-lived stimulation equilibria exist only for $p_g < 0.804$, which is well below the upper bound $2\theta_0 - \mathbb{E}[\theta | \theta \leq \theta_0] = 1$ identified in Theorem 3-(iii). Although not seen in the figure, there are typical multiple delayed equilibria (when at least one exists), but all of them induce the same threshold $\hat{\theta} = \theta_0$ and the same total trade volume,



(a) $\hat{\theta}$'s in short-lived stimulation equilibria (b) Overall trade in both types of equilibria

Figure 9 – Equilibrium outcome in the uniform example ($S = 1/3, I = 1/10$)

as formally stated in Theorem 4.

Second, both types of equilibria coexist for a range of p_g 's, as can be seen in the figure. This multiplicity reflects the endogenous nature of equilibrium belief formation. Given a p_g , suppose a large measure of high-type firms accept a bailout but boycott the $t = 2$ market to avoid stigma. This causes buyers to adjust down their offers in the $t = 2$ market, in turn validating the firms' decision not to hold out in $t = 1$. A short-lived stimulation equilibrium then arises. By contrast, if a large measure of high-type firms hold out in $t = 1$ for the same p_g , then buyers in $t = 2$ make high offers to attract them, which in turn validates their decision to hold out, leading to a delayed stimulation equilibrium.

Third, as can be seen from Figure 9-(b), the effect of bailout can be discontinuous with respect to the terms of bailout. Suppose the policy maker raises the bailout term p_g starting from a low value close to $p_0 = 1/3$. At first, a short-lived stimulation equilibria may arise. As p_g rises past 0.804, however, short-lived stimulation equilibria disappear and in the resulting delayed stimulation equilibrium, the total trade volume jumps up albeit with delay. As mentioned before, one policy implication is that the policy maker may need to choose an attractive bailout offer ($p_g > 0.804$ in this example) in order to avoid the selection of less desirable short-lived stimulation equilibria.

Lastly, despite the stigma, a bailout with $p_g > p_0$ boosts overall trade regardless of the types of equilibria selected. As shown in Figure 9-(b), the total trade volume in short-lived

stimulation equilibrium is strictly higher than $2F(\theta_0) = 4/3$, the total trade volume in the benchmark without bailout. This suggests that the positive effect of bailout on asset trading in $t = 1$ —i.e., every type $\theta \leq p_g + S$ sells in $t = 1$ —outweighs the negative effect of stigma on asset trading in $t = 2$. Further, as can be seen in Figure 9-(b), the total trade volume in short-lived stimulation equilibrium tends to increase with p_g , so a more generous bailout tends to have a higher stimulation effect.

5 Cost of Bailouts and Welfare

In the preceding analysis, we have abstracted from the cost of bailouts although it is a crucial part of policy debates. In this section, we evaluate alternative bailout equilibria and characterize the optimal bailout term p_g , with the cost of bailouts explicitly accounted for. To model the cost, we follow the literature (Tirole, 2012; Chiu and Koepl, 2016) and assume that raising a dollar of public funds used for the asset purchase program costs the society $(1 + \lambda)$ dollars, where $\lambda \geq 0$ represents the deadweight loss of raising public funds, e.g., distortionary taxation. The welfare effect of a bailout policy would be then captured by the investment surplus generated by the policy minus the total cost of raising public funds that the policy would incur.

To evaluate the welfare of an equilibrium resulting from a bailout policy, it is convenient to adopt a mechanism design perspective. In particular, we represent the **outcome** of an equilibrium by a pair of mappings, $(Q, T) : [0, 1] \rightarrow \{0, 1, 2\} \times \mathbb{R}$, induced by that equilibrium, where $Q(\theta) \in \{0, 1, 2\}$ is a type- θ firm's total asset sale across the two periods and $T(\theta)$ is the total transfer it receives across the two periods in equilibrium. The transfer includes payment from both the government (if the firm accepts a bailout) and private buyers (if it sells to the market in either period). For our analysis we can view these mappings as a *direct mechanism* that implements a social choice as a function of report on the firm's type θ . For this mechanism to represent an equilibrium outcome, it must then satisfy the usual incentive compatibility and participation conditions. One can then invoke the celebrated *revenue equivalence* or envelope theorem to characterize the welfare of a particular equilibrium via the trade volume and a payoff for a reference type (e.g., the highest type) induced by the equilibrium.

The next lemma provides this characterization.

Lemma 2. *Any equilibrium outcome (Q, T) arising from a bailout policy yields welfare:*

$$\int_0^1 \left\{ 2\theta + SQ(\theta) - \lambda \left[\left(\frac{F(\theta)}{f(\theta)} - S \right) Q(\theta) + \bar{u} - 2 \right] \right\} f(\theta) d\theta, \quad (7)$$

where $\bar{u} \geq 2$ is the highest-type ($\theta = 1$) firm's payoff across the two periods in the equilibrium.

Proof. See Appendix A.

Q.E.D.

The welfare of an equilibrium outcome consists of three terms. The first term, 2θ , is simply the value of type- θ asset; recall that each firm owns 2 units of such asset, and its value does not depend on who eventually owns that asset. The second term, $SQ(\theta)$, corresponds to the return from the project enabled by the sale of type- θ asset represented by $Q(\theta)$. The last term accounts for the cost of bailouts. In particular, the terms inside the square brackets correspond to the budget shortfall arising from the equilibrium.²⁶ Recall that private buyers break even in expectation in any equilibrium. Hence, each of this budget shortfall must be paid for by public funds and thus incurs the social cost of λ . The budget shortfall consists of three terms. The term, $\bar{u} - 2$, is the rent that the highest-type ($\theta = 1$) firm enjoys in equilibrium above its asset value 2. (By the standard envelope theorem reasoning, this rent must accrue to “all” firm types.) The next term, $\frac{F(\theta)}{f(\theta)}Q(\theta)$, is the incentive cost that is required to induce type- θ firm to sell additional unit of its asset: since any sale by type θ can be mimicked profitably by all lower types, these types must be paid rents to prevent their mimicking. Since higher types can be mimicked by more types, the incentive cost is increasing in θ . Last, project returns $SQ(\theta)$ act as an incentive for sale by type θ : they mitigate the budget shortfall and save the subsidy needed to induce a sale.

The fact that sale by a higher type incurs a higher social cost provides a key argument for comparing equilibrium outcomes. To see this, fix an equilibrium, say A , with allocation $Q_A(\cdot)$ resulting from a bailout at p_g . Suppose the policy maker, with some bailout term $p'_g \neq p_g$, can trigger another equilibrium, say B , with allocation $Q_B(\cdot)$, such that (a) the aggregate trade volume remains unchanged; i.e., $\mathbb{E}[Q_A(\theta)] = \mathbb{E}[Q_B(\theta)]$ but that (b) B “reallocates” sales away from high-type firms towards low-type firms; i.e., $Q_B(\theta) \geq Q_A(\theta)$ if $\theta \leq \check{\theta}$ for some $\check{\theta}$. Then, a shift from A to B preserves the same stimulation effect, and thus the same investment returns, but incurs a lower budget deficit borne by the government, and thus a lower social cost.

This reasoning establishes the social cost of bailout stigma as follows.

Theorem 5 (Welfare cost of bailout stigma). *Fix any bailout offer p_g that yields a short-lived stimulation equilibrium under a transparent bailout. Then, there exists a $p'_g < p_g$ such that a secret bailout with p'_g yields strictly higher welfare than the short-lived stimulation equilibrium.*

Proof. See Appendix A.

Q.E.D.

²⁶Again, letting Θ_g denote the types of firms that accept the bailout, the government suffers a loss of

$$\int_{\Theta_g} (p_g - \theta)f(\theta)d\theta = \int_0^1 (T(\theta) - \theta Q(\theta))f(\theta)d\theta = \bar{u} - 2 + \int_0^1 \left(\frac{F(\theta)}{f(\theta)} - S \right) Q(\theta)f(\theta)d\theta,$$

where we used the envelope theorem to substitute for $T(\theta)$.

Intuitively, the additional welfare cost of a short-lived stimulation equilibrium is attributed to the overall market-freezing effect caused by bailout stigma. In the equilibrium, bailout stigma discourages firms not only from accepting a bailout but also from selling to the market in $t = 1$. This means that, in order to induce the same trade volume as under a secret bailout, the government must raise its bailout term p_g , which leads to a higher cost of public intervention.

Given the outcome equivalence between a delayed stimulation equilibrium under transparent bailout and the equilibrium under secret bailout (Corollary 1), the following corollary is immediate.

Corollary 2. *A delayed stimulation equilibrium given any bailout term p_g yields identical welfare as a secret bailout given the same bailout term.*

6 Related Literature

While the broad theme of this paper is related to an extensive literature on the benefits and costs of government intervention in distressed banks,²⁷ our work is most closely related to Philippon and Skreta (2012) and Tirole (2012), who focus on adverse selection in asset markets as a primary reason for government intervention.²⁸ As mentioned previously, these studies do not explicitly study the dynamic consequence of receiving a bailout—the focus of the current study. Even though these papers recognize that relatively low types accept bailouts, this does not translate into an adverse effect on subsequent financing in their models. Our dynamic model captures not only how bailout stigma affects firms’ financing behavior but also how the stigma fundamentally alters the role of a bailout. In particular, its role in enabling firms to send a favorable signal by “refusing” an attractive bailout offer is the single most striking takeaway that has no analogues in these or other antecedent studies.

Banks’ reputational concerns are explicitly considered in Ennis and Weinberg (2013), La’O (2014), Chari, Shourideh and Zetlin-Jones (2014), and Ennis (2019). In Ennis and Weinberg (2013), to meet their short-term liquidity needs, banks with high-quality assets use interbank

²⁷The primary rationale for intervention is to prevent the contagion of bank runs whether it stems from depositor panic (Diamond and Dybvig, 1983), contractual linkages in bank lending (Allen and Gale, 2000), or aggregate liquidity shortages (Diamond and Rajan, 2005). The costs of anticipated bailouts due to the time-inconsistency of policy are discussed by, among others, Stern and Feldman (2004).

²⁸Regarding the optimal form of bailouts, Philippon and Skreta (2012) show that optimal interventions involve the use of debt instruments when adverse selection is the main issue. With additional moral hazard but limits on pledgeable income, Tirole (2012) justifies asset purchases. When there is debt overhang due to lack of capital, Philippon and Schnabl (2013) find that optimal interventions take the form of capital injection in exchange for preferred stock and warrants. During the US subprime crisis, the EESA initially granted the Secretary of the Treasury authority to purchase or insure troubled assets owned by financial institutions. However, the Capital Purchase Program under TARP switched to capital injection against preferred stock and warrants.

lending while those with low-quality assets use the discount window. The resulting discount window stigma is reflected in the subsequent pricing of assets. In [La'O \(2014\)](#), financially strong banks use the Federal Reserve's Term Auction Facility since winning the auction at a premium signals financial strength, which protects them from predatory trading. The main focus in [Chari, Shourideh and Zetlin-Jones \(2014\)](#) is on how reputational concerns in secondary loan markets can result in persistent adverse selection. Since all three studies consider discrete types of banks and there is no government bailout, their results are not directly comparable to ours. [Ennis \(2019\)](#) extends [Philippon and Skreta \(2012\)](#) by allowing banks to borrow from the discount window before borrowing from the market, thereby formalizing the discount window stigma alluded to in [Philippon and Skreta \(2012\)](#). There are two main differences between these studies and our work. First, there is only one investment opportunity in these studies; hence, they do not capture the dynamics of a stimulation effect—e.g., the possibility of stimulation being either short-lived or delayed. Second, they characterize equilibria for given discount window rates or the auction mechanism, whereas we study the optimal bailout policy. The main focus in [Chari, Shourideh and Zetlin-Jones \(2014\)](#) is on how reputational concerns in secondary loan markets can result in persistent adverse selection, but they do not consider government bailout.

Our paper is also related to studies on dynamic adverse selection in general ([Inderst and Müller, 2002](#); [Janssen and Roy, 2002](#); [Moreno and Wooders, 2010](#); [Camargo and Lester, 2014](#); [Fuchs and Skrzypacz, 2015](#)) and those with a specific focus on the role of information in particular ([Hörner and Vieille, 2009](#); [Daley and Green, 2012](#); [Fuchs, Öry and Skrzypacz, 2016](#); [Kim, 2017](#)).²⁹ The key insight from the first set of studies is that dynamic trading generates sorting opportunities, which are not available in the static market setting. However, each seller has only one opportunity to trade in these studies, so signaling is not an issue. The second set of studies relates to different disclosure rules and how they affect dynamic trading. For example, [Hörner and Vieille \(2009\)](#) and [Fuchs, Öry and Skrzypacz \(2016\)](#) show that secrecy (private offers) tends to alleviate adverse selection but transparency (public offers) does not. Once again, each seller has only one trading opportunity in these studies. Hence, although past rejections can boost reputation, acceptance ends the game. In contrast, in our model, acceptance as well as rejection of the bailout offer work as signaling opportunities. Although our model also shows that secret bailouts weakly dominate transparent bailouts, none of these papers studies government intervention in response to market failure.

There are several empirical studies that provide evidence on stigma in the financial market. [Peristiani \(1998\)](#) provides early evidence on the discount window stigma. [Furfine \(2001, 2003\)](#) finds similar evidence from the Federal Reserve's Special Lending Facility during the period 1999-2000 and the new discount window facility introduced in 2003. As mentioned earlier,

²⁹Others include dynamic extensions of Spence's signaling model with public offers ([Noldeke and Van Damme, 1990](#)), private offers ([Swinkels, 1999](#)), and private offers with additional public information such as grades ([Kremer and Skrzypacz, 2007](#)).

[Armantier et al. \(2015\)](#) utilize the Federal Reserve’s Term Auction Facility bid data from the 2007-2008 financial crisis to estimate the cost of stigma and its effect. [Cassola, Hortag̃su and Kastl \(2013\)](#) find evidence of stigma from the bidding data from the European Central Bank’s auctions of one-week loans. [Krishnamurthy, Nagel and Orlov \(2014\)](#) find that in repo financing, dealer banks with higher shares of agency collateral repayments (implicitly) guaranteed by the government borrowed less from the Primary Dealer Credit Facility (PDCF) despite its attractive funding terms, which indeed supports there being a stigma attached to the users of the PDCF.

7 Conclusion

The current paper has studied a dynamic model of a government bailout in which firms have a continuing need to fund their projects by selling their assets. Asymmetric information about the quality of assets gives rise to adverse selection and a concomitant market freeze, which provides a rationale for a government bailout, just as in [Tirole \(2012\)](#). However, in contrast to [Tirole \(2012\)](#), markets stigmatize bailout recipients, which jeopardizes their ability to fund subsequent projects. The presence of this bailout stigma and other dynamic incentives yields a much more complex and nuanced portrayal of how bailouts impact the economy than have been recognized in the extant literature.

Our main findings can be summarized as follows. The bailout stigma necessitates the government to pay a premium over the market terms to compensate for the stigma. Even so, market rejuvenation can be short-lived and adverse selection can worsen in subsequent market trading, resulting in a market freeze even more severe than in the absence of a bailout. This requires the government to further increase a premium. A more attractive bailout premium can be effective in stimulating trade and investment, but its effects are delayed. Delayed benefits materialize as bailouts provide firms with opportunities to boost reputation by “rejecting” bailout offers. This improves their ability to trade in the market in subsequent periods. Indeed, there is no welfare loss in this case relative to a secret bailout that does not entail stigma. Thus delayed effects of bailouts can be a blessing in disguise, subject to two important caveats: the government may need to run a large budget deficit to support delayed market stimulation, and delay in and of itself may be undesirable for reasons not modeled in the current paper.

The central lesson from the current work is that, compared with the static setting, the effects of bailouts are very different due to the interplay between the bailout stigma, the market’s belief within and across periods, and rich signaling opportunities firms have in the dynamic context. To the best of our knowledge, the insights we develop and the forces we identify are novel and have not been recognized in the previous literature and should be part of the framework for conducting future policy debates and empirical studies.

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Appendix

A Proofs

A.1 Proof of Theorem 3

Proof of (i): $\bar{\theta}_g < \bar{\theta}_m < p_g$. We first establish the following claim.

Claim 1. $\bar{\theta}_g < \bar{\theta}_m$.

Proof: Suppose to the contrary that $\bar{\theta}_g \geq \bar{\theta}_m$, which in turn implies $p_g \leq \bar{\theta}_m$ from (4). Given Lemma 1, there are two possible cases to consider: $\hat{\theta} < \hat{\theta}_g$ or $\hat{\theta} = \hat{\theta}_g$.

Suppose first $\hat{\theta} < \hat{\theta}_g$. Then, we have $\hat{\theta}_g \leq p_g + S$ from (6). Moreover, we have from (3) that $\bar{\theta}_m \leq \mathbb{E}[\theta | \theta \leq \hat{\theta}] \leq \mathbb{E}[\theta | \theta \leq p_g + S]$. Since $p_g > p_0$, we must have $\mathbb{E}[\theta | \theta \leq p_g + S] < p_g$, or else $p_g \leq p_0$ (recall definition of p_0 from (1) as well as Figure fig:akerlof). Hence, $\bar{\theta}_m < p_g$, which, however, contradicts the earlier hypothesis $p_g \leq \bar{\theta}_m$.

Suppose next $\hat{\theta} = \hat{\theta}_g$. Then, a type- $\hat{\theta}$ firm must be indifferent between selling in both periods and selling in neither period if $\hat{\theta} < 1$ and (weakly) prefer selling in both periods

if $\hat{\theta} = 1$. Hence, we must have $2\hat{\theta} \leq 2\bar{\theta}_m + 2S$, where the inequality holds strictly only if $\hat{\theta} = 1$. We thus conclude that $\hat{\theta} = (\bar{\theta}_m + S) \wedge 1$. Moreover, we have from (3) that $\bar{\theta}_m \leq \mathbb{E}[\theta|\theta \leq \hat{\theta}] \leq \mathbb{E}[\theta|\theta \leq \bar{\theta}_m + S]$, which implies $\bar{\theta}_m \leq p_0$ (again by the definition of θ_0 in (1)). Since $p_g > p_0$, this again contradicts an earlier hypothesis that $p_g \leq \bar{\theta}_m$. We thus conclude that $\bar{\theta}_g < \bar{\theta}_m$. \square .

By (4), Claim 1 in turn implies that $\bar{\theta}_m < p_g$. We have thus proven (i). \square

Proof of (ii): $\hat{\theta} < \theta_0 < \hat{\theta}_g = p_g + S$. We first establish the following two claims:

Claim 2. $\hat{\theta} \neq 1$.

Proof: Suppose to the contrary that $\hat{\theta} = 1$. Then, $\hat{\theta} = 1 \leq \bar{\theta}_g + S$. Otherwise, we will have $p_g + S + \hat{\theta} > p_g + \bar{\theta}_g + 2S = 2\bar{\theta}_m + 2S$, where the equality is from (4), so a type- $\hat{\theta}$ firm would deviate by accepting the bailout but boycotting the $t = 2$ market. Since $\bar{\theta}_g < \bar{\theta}_m$ (by Claim 1) and $\mu_g \bar{\theta}_g + \mu_m \bar{\theta}_m = \mathbb{E}[\theta|\theta \leq \hat{\theta}] = \mathbb{E}[\theta|\theta \leq 1]$, we have

$$1 \leq \bar{\theta}_g + S < \mu_g \bar{\theta}_g + \mu_m \bar{\theta}_m + S \leq \mathbb{E}[\theta|\theta \leq 1] + S.$$

But this contradicts $\theta_0 < 1$, which we assume throughout. \square

Claim 3. $\hat{\theta} < \hat{\theta}_g$.

Proof: Suppose to the contrary that $\hat{\theta} = \hat{\theta}_g$. By Claim 2, we have $\hat{\theta} = (\bar{\theta}_m + S) \wedge 1$ and $p_g > \bar{\theta}_m$. In equilibrium, each type $\theta > \hat{\theta}$ never sells in either period and obtains payoff 2θ . Since $p_g > \bar{\theta}_m$, however, types $\theta \in (\hat{\theta}_g, p_g + S)$ will have a strictly higher payoff than 2θ by selling to the government in $t = 1$, a contradiction. \square

We are now ready to prove (ii). We first show that $\hat{\theta}_g > \theta_0$. Since $p_g > p_0$, it is straightforward from (6) that $\hat{\theta}_g = (p_g + S) \wedge 1 > \theta_0$. We next prove $\hat{\theta} < \theta_0$. Since $\hat{\theta} < \hat{\theta}_g$ by Claim 3, we have

$$\hat{\theta} = \bar{\theta}_g + S < \mathbb{E}[\theta|\theta \leq \hat{\theta}] + S,$$

where the equality follows from (4) and (5), and the strict inequality follows from $\bar{\theta}_g < \bar{\theta}_m$ and (3). The definition of θ_0 then implies $\hat{\theta} < \theta_0$. \square

Proof of (iii): *a short-lived stimulation equilibrium exists only if $p_g < 2\theta_0 - \mathbb{E}[\theta|\theta \leq \theta_0]$.* To prove this, observe from (4) that

$$p_g = 2\bar{\theta}_m - \bar{\theta}_g.$$

Fixing $\hat{\theta}$, the RHS is maximized when, for some threshold $\tilde{\theta} \in [0, \hat{\theta}]$, all types $\theta > \tilde{\theta}$ sell to the market and all types $\theta < \tilde{\theta}$ sell to the government so that $\bar{\theta}_m = \mathbb{E}[\theta|\theta \in (\tilde{\theta}, \hat{\theta}]$ and

$\bar{\theta}_g = \mathbb{E}[\theta|\theta \leq \tilde{\theta}]$. Hence,

$$\begin{aligned}
p_g &= 2\bar{\theta}_m - \bar{\theta}_g \\
&\leq \max_{\tilde{\theta} \in [0, \hat{\theta}], \theta \in [0, \theta_0]} 2\mathbb{E}[\theta|\theta \in (\tilde{\theta}, \hat{\theta}]] - \mathbb{E}[\theta|\theta \leq \tilde{\theta}] \\
&< \max_{\tilde{\theta} \in [0, \theta_0]} 2\mathbb{E}[\theta|\theta \in (\tilde{\theta}, \theta_0]] - \mathbb{E}[\theta|\theta \leq \tilde{\theta}] \\
&= 2\theta_0 - \mathbb{E}[\theta|\theta \leq \theta_0],
\end{aligned}$$

where the strict inequality follows from $\hat{\theta} < \theta_0$ and $\mathbb{E}[\theta|\theta \in (a, b)]$ is increasing in b for all $0 \leq a \leq b \leq 1$, and the last equality follows from the regularity condition that $2\mathbb{E}[\theta|\theta \in (a, b)] - \mathbb{E}[\theta|\theta \leq a]$ is increasing in a for all $0 \leq a \leq b \leq 1$. \square

A.2 Proof of Theorem 4

As before, let μ_g and μ_m respectively denote the fractions of types $\theta \leq \hat{\theta}$ that sell to the government and to the market in $t = 1$, and let $\bar{\theta}_g$ and $\bar{\theta}_m$ denote their average values. Obviously, (3) must continue to hold. Let p_m be the market price for the asset in $t = 1$, and let p_2^g and p_2^m respectively denote the $t = 2$ prices for those that sold to the government and those that did not. Note that p_2^m applies to those that sold to the market in $t = 1$ and to those that held out, since $t = 2$ cannot distinguish them.

We wish to prove that $\hat{\theta} = \theta_0$, $\theta_2 = p_g + S$, and $\mu_g > 0$. There are two possible cases: $\hat{\theta}_g > \hat{\theta}$ and $\hat{\theta}_g = \hat{\theta}$, and we treat them separately. (Recall by definition $\hat{\theta}_g \geq \hat{\theta}$.)

A.2.1 The case of $\hat{\theta}_g > \hat{\theta}$.

In this case, firms with $\theta \in (\hat{\theta}, \hat{\theta}_g]$ sell to the government in $t = 1$. Obviously, $\mu_g \geq \int_{\hat{\theta}}^{\hat{\theta}_g} f(\theta)d\theta > 0$. We only need to prove $\hat{\theta} = \theta_0$ and $\theta_2 = p_g + S$.

We first show $\theta_2 = p_g + S$. Observe that a type- $\hat{\theta}_g$ firm must be indifferent between selling only in $t = 1$ to the government at p_g and selling only in $t = 2$ at price p_2^m . Hence, we must have $p_2^m = p_g$. Since a type- θ_2 firm must be indifferent between selling only in $t = 2$ at price p_2^m and not selling in any period, we must have $\theta_2 = p_2^m + S = p_g + S$.

We next show $\hat{\theta} = \theta_0$. We restrict our focus on the case $\mu_g > 0$ and $\mu_m > 0$ (The argument for the other case $\mu_m = 0$ is similar, so we omit the proof). Then, these firms sell in both periods, and thus must be indifferent between selling to the government and to the market in $t = 1$. This implies (4), or

$$p_g + p_2^g = p_m + p_2^m = p_m + p_g \Rightarrow p_m = p_2^g.$$

This, together with the zero-profit condition, implies that

$$\bar{\theta}_g = p_2^g = p_m = \bar{\theta}_m. \quad (8)$$

It then follows from (3) that

$$\bar{\theta}_g = \bar{\theta}_m = \mathbb{E}[\theta | \theta \leq \hat{\theta}]. \quad (9)$$

Next, a type- $\hat{\theta}$ firm must be indifferent between selling in both periods and selling only in $t = 1$ to the government at price p_g :

$$p_m + p_g + 2S = p_g + S + \hat{\theta} \iff p_m + S = \hat{\theta}, \quad (10)$$

which, together with (8) and (9), implies that

$$\mathbb{E}[\theta | \theta \leq \hat{\theta}] + S = \hat{\theta}.$$

By definition of θ_0 , or (1), we then have $\hat{\theta} = \theta_0$. This in turn implies $p_m = p_2^g = \mathbb{E}[\theta | \theta \leq \theta_0] = p_0$.

A.2.2 The case of $\hat{\theta}_g = \hat{\theta}$.

The proof proceeds in several claims.

Claim 4. $\mu_g > 0$.

Proof: Suppose to the contrary that $\mu_g = 0$. Then, we must have $p_2^m = p_0$ since buyers in $t = 2$ do not observe any action taken by firms in $t = 1$. This implies $p_0 = p_2^m = p_g > p_0$, a contradiction. \square

Claim 5. $\theta_2 = p_g + S$.

Proof: Since types $\theta \in (\hat{\theta}, \theta_2)$ must weakly prefer selling only in $t = 2$ at price p_2^m to selling only in $t = 1$ to the government at price p_g . This implies $p_2^m \geq p_g$. We now prove that $p_2^m = p_g$. Suppose to the contrary that $p_2^m > p_g$.

We know from Claim 4 that $\mu_g > 0$. Suppose $\mu_m > 0$. No arbitrage between selling to the government and selling to the market in $t = 1$ means that $p_g + p_2^g = p_m + p_2^m$, so we have

$$p_2^g = (p_2^m - p_g) + p_m > p_m, \quad (11)$$

where the strict inequality follows from our hypothesis above that $p_2^m > p_g$. Furthermore, since type $\hat{\theta}$ must be indifferent between selling in both periods and selling only in $t = 2$ at p_2^m , we must have

$$\hat{\theta} + p_2^m + S = p_m + p_2^m + 2S \implies \hat{\theta} = p_m + S. \quad (12)$$

By the zero profit condition, $p_2^g = \bar{\theta}_g$ and $p_m = \bar{\theta}_m$. Hence, by (11), we have $\bar{\theta}_g > \bar{\theta}_m$. By (3), we must have

$$\mathbb{E}[\theta|\theta \leq \hat{\theta}] > \bar{\theta}_m = p_m. \quad (13)$$

Then (13) and (12) imply

$$\mathbb{E}[\theta|\theta \leq \hat{\theta}] + S > \hat{\theta}. \quad (14)$$

This means that $\hat{\theta} < \theta_0$, by the definition of θ_0 . By (13), this means that

$$p_m = \bar{\theta}_m < \mathbb{E}[\theta|\theta \leq \hat{\theta}] < \mathbb{E}[\theta|\theta \leq \theta_0] = p_0,$$

where the last equality follows from the definition of p_0 . Suppose a buyer deviates and offers $p' \in (p_m, p_0)$. Since $p' + p_2^m > p_m + p_2^m = p_g + p_2^g$ for any $p' \in (p_m, p_0)$, all types $\theta \leq \hat{\theta}$ will sell to this deviating buyer. Furthermore, since $p_g + S + \theta \leq p' + S + p_2^m + S \implies \theta \leq p' + S$ for any $p' \in (p_m, p_0)$, types $\theta \in (\hat{\theta}, p' + S]$ will sell at the deviation price, too. Since $p' < p_0$, we have $\mathbb{E}[\theta|\theta \leq p' + S] - p' > 0$, so the deviating buyer will enjoy a strict profit. We have thus obtained a contradiction to the hypothesis that $p_2^m > p_g$. A similar conclusion is also obtained when $\mu_m = 0$.³⁰ We therefore conclude that $p_2^m = p_g$. Since $\theta_2 = p_2^m + S$, this in turn implies that $\theta_2 = p_g + S$. \square

Claim 6. $\hat{\theta} = \theta_0$.

Proof: We know from Claim 4 that $\mu_g > 0$. There are two cases depending on whether $\mu_m > 0$ or $\mu_m = 0$. Consider the former first. No arbitrage for type $\theta \in [0, \hat{\theta}]$ between selling to the government and selling to the market in $t = 1$ implies $p_g + p_2^g = p_m + p_2^m$, which in turn implies $p_2^g = p_m$ since $p_g = p_2^m$. By the zero-profit condition, $p_2^g = \bar{\theta}_g$ and $p_m = \bar{\theta}_m$, so by the zero-profit condition $\bar{\theta}_g = \bar{\theta}_m$. Hence, by (3), we get $p_m = \mathbb{E}[\theta|\theta \leq \hat{\theta}]$. Combining this equality with (12), we have

$$\hat{\theta} = \mathbb{E}[\theta|\theta \leq \hat{\theta}] + S.$$

By definition of θ_0 , the above equality implies $\hat{\theta} = \theta_0$.

Consider next $\mu_m = 0$. Type $\hat{\theta}$ must be indifferent now between selling to the government in $t = 1$ followed by selling to the $t = 2$ market (with stigma) and selling only in $t = 2$ market. Hence,

$$p_2^g + p_g + 2S = \hat{\theta} + p_m^2 + S. \quad (15)$$

³⁰In this case, again the indifference for type $\hat{\theta}$ gives $p_g + \mathbb{E}[\theta|\theta \leq \hat{\theta}] + S = \hat{\theta} + p_2^m$. Since $p_2^m > p_g$, this implies $\hat{\theta} < \theta_0$, or $\mathbb{E}[\theta|\theta \leq \hat{\theta}] < p_0$. This creates an opportunity for buyers to profitably deviating by offering a price $p'_m \in (\mathbb{E}[\theta|\theta \leq \hat{\theta}], p_0)$.

In the proof of Claim 5, we already proved that $p_m^2 = p_g$. Hence, (15) reduces to

$$p_2^g + S = \hat{\theta}. \quad (16)$$

Further, by the zero profit condition,

$$p_2^g = \bar{\theta}_m = \mathbb{E}[\theta | \theta < \hat{\theta}], \quad (17)$$

where the second equality holds since $\mu_m = 0$. Combining (16) and (17) gives

$$\hat{\theta} = \mathbb{E}[\theta | \theta < \hat{\theta}] + S,$$

which proves that $\hat{\theta} = \theta_0$. \square

A.3 Proof of Lemma 2

Fix a direct mechanism (Q, T) . Define $u(\tilde{\theta}|\theta) := T(\tilde{\theta}) + \theta(2 - Q(\tilde{\theta})) + SQ(\tilde{\theta})$ as type- θ firm's payoff when it reports its type as $\tilde{\theta}$. Since the mechanism must be incentive compatible for all types, we have $u(\theta|\theta) \geq u(\tilde{\theta}|\theta)$ for all $\tilde{\theta}, \theta \in [0, 1]$. Let $u(\theta) := u(\theta|\theta)$. Since the participation constraint must be satisfied for all types, we also have $u(\theta) \geq 2\theta$ for all $\theta \in [0, 1]$. Let $\bar{u} = u(1)$ be the highest-type ($\theta = 1$) firm's payoff in equilibrium. Then, one can apply the envelope theorem to find $u(\theta) = \bar{u} - \int_{\theta}^1 (2 - Q(s)) ds$.

To calculate the welfare, let Θ_g denote the set of θ 's that sell to the government in $t = 1$. The welfare is the sum of the payoffs for firms and buyers minus the deadweight loss from a deficit run by the government. Given the government offer p_g , the welfare is then written as $\int_0^1 [u(\theta) + (\theta Q(\theta) - T(\theta))] f(\theta) d\theta - \lambda \int_{\theta \in \Theta_g} (p_g - \theta) f(\theta) d\theta$. Since buyers in the market must break even, we have $\int_{\theta \in \Theta_g} (p_g - \theta) f(\theta) d\theta = \int_0^1 (T(\theta) - \theta Q(\theta)) f(\theta) d\theta$. Hence, the welfare is $\int_0^1 [u(\theta) + (\theta Q(\theta) - T(\theta)) - \lambda(T(\theta) - \theta Q(\theta))] f(\theta) d\theta$. By plugging $u(\theta) = \bar{u} - \int_{\theta}^1 (2 - Q(s)) ds$ into the welfare and integrating by parts, we obtain (7).

A.4 Proof of Theorem 5

Consider a transparent bailout with p_g that yields a short-lived stimulation equilibrium and let (Q_{SL}, T_{SL}) denote the corresponding outcome (expressed in a direct mechanism). Then, we have $Q_{SL}(\theta) = 2$ if $\theta \in [0, \hat{\theta}]$, $Q_{SL}(\theta) = 1$ if $\theta \in (\hat{\theta}, (p_g + S) \wedge 1]$, and $Q_{SL}(\theta) = 0$ otherwise. Hence, the total trade volume induced by the short-lived stimulation equilibrium is $F(\hat{\theta}) + F((p_g + S) \wedge 1)$. Furthermore, the utility of the highest-type firm, denoted by \bar{u}_{SL} , is equal to $1 + ((p_g + S) \vee 1)$. Next, consider an equilibrium under a secret bailout with p'_g and let (Q_S, T_S) denote the corresponding outcome. Then we have $Q_S(\theta) = 2$ if $\theta \in [0, \theta_0]$,

$Q_S(\theta) = 1$ if $\theta \in (\theta_0, (p'_g + S) \wedge 1]$, and $Q_S(\theta) = 0$ otherwise. Thus, the total trade volume in this case is $F(\theta_0) + F((p'_g + S) \wedge 1)$, and the utility of the highest-type firm, denoted by \bar{u}_S , is equal to $1 + ((p_g + S) \vee 1)$. Suppose the secret bailout with p'_g yields the same total trade volume as the short-lived stimulation equilibrium under the transparent bailout with p_g . Since $\hat{\theta} < \theta_0$ from Theorem 3, we must have $(p'_g + S) \wedge 1 < (p_g + S) \wedge 1$, which implies $p'_g < p_g$. This also implies $\bar{u}_S = 1 + ((p'_g + S) \vee 1) \leq 1 + ((p_g + S) \vee 1) = \bar{u}_{SL}$.

We next show that the secret bailout with p'_g yields strictly higher welfare than the short-lived stimulation equilibrium arising from p_g , if the secret bailout induces the same total trade volume as the short-lived stimulation equilibrium does. From (7), the welfare difference between the two types of equilibria is

$$\begin{aligned}
& \int_0^1 \left\{ S(Q_S(\theta) - Q_{SL}(\theta)) - \lambda \left[\left(\frac{F(\theta)}{f(\theta)} - S \right) (Q_S(\theta) - Q_{SL}(\theta)) + (\bar{u}_S - \bar{u}_{SL}) \right] \right\} f(\theta) d\theta \\
&= (1 + \lambda)S \int_0^1 (Q_S(\theta) - Q_{SL}(\theta)) f(\theta) d\theta + \lambda \int_0^1 \left[\frac{F(\theta)}{f(\theta)} (Q_{SL}(\theta) - Q_S(\theta)) + (\bar{u}_{SL} - \bar{u}_S) \right] f(\theta) d\theta \\
&= \lambda \int_0^1 \left[\frac{F(\theta)}{f(\theta)} (Q_{SL}(\theta) - Q_S(\theta)) + (\bar{u}_{SL} - \bar{u}_S) \right] f(\theta) d\theta \\
&\geq \lambda \int_0^1 \frac{F(\theta)}{f(\theta)} (Q_{SL}(\theta) - Q_S(\theta)) f(\theta) d\theta \\
&> \lambda \left[\frac{F(\theta_0)}{f(\theta_0)} \int_0^{\theta_0} (Q_{SL}(\theta) - Q_S(\theta)) f(\theta) d\theta + \frac{F(\theta_0)}{f(\theta_0)} \int_{\theta_0}^1 (Q_{SL}(\theta) - Q_S(\theta)) f(\theta) d\theta \right] \\
&= \lambda \frac{F(\theta_0)}{f(\theta_0)} \int_0^1 (Q_{SL}(\theta) - Q_S(\theta)) f(\theta) d\theta = 0,
\end{aligned}$$

where the second and last equalities follow from $\int_0^1 (Q_S(\theta) - Q_{SL}(\theta)) f(\theta) d\theta = 0$, the weak inequality follows from $\bar{u}_{SL} \geq \bar{u}_S$, and the strict inequality follows from the facts that $\frac{F(\theta)}{f(\theta)}$ is strictly increasing in θ , $\int_0^{\theta_0} (Q_{SL}(\theta) - Q_S(\theta)) f(\theta) d\theta < 0$, and $\int_{\theta_0}^1 (Q_{SL}(\theta) - Q_S(\theta)) f(\theta) d\theta > 0$.

Online Appendix

B Possible Types of Equilibria

In this section, we show that there are only two types of equilibria, short-lived stimulation and delayed stimulation types. To this end, recall first from Lemma 1 that every equilibrium must have a cutoff structure $0 \leq \hat{\theta} \leq \hat{\theta}_g \leq \theta_2 \leq 1$ such that firms with types $0 \leq \hat{\theta}$ sell in both periods (either to the government or to the market in $t = 1$); types $\theta \in (\hat{\theta}, \hat{\theta}_g]$ sell in $t = 1$ but not in $t = 2$; types $\theta \in (\hat{\theta}_g, \theta_2]$ sell only in $t = 2$; types $\theta > \theta_2$ do not sell in either period.

Lemma 3. *In the presence of government bailouts, there are only two possible types of equilibria, $0 < \hat{\theta} < \hat{\theta}_g = \theta_2 \leq 1$ (short-lived stimulation type) or $0 < \hat{\theta} \leq \hat{\theta}_g < \theta_2 \leq 1$ (delayed stimulation type).*

Proof. For the purpose of exposition, let p_m , p_2^g , and p_2^m denote equilibrium asset prices offered to all firms at the market in $t = 1$, the bailout recipients (i.e., the firms that sold to the government in $t = 1$) in $t = 2$, and the bailout holdouts (i.e., the firms that did not sell to the government in $t = 1$) in $t = 2$, respectively. Note that there are no firms that sell to the market in $t = 1$ but not in $t = 2$: if there were such firms, buyers in $t = 2$ would offer p_m to attract these firms.

It suffices to prove that $\hat{\theta} > 0$ and $\hat{\theta} < \max\{\hat{\theta}_g, \theta_2\}$. We prove these claims in sequence.

Step 1. $\hat{\theta} > 0$.

Suppose to the contrary that there exists an equilibrium with $\hat{\theta} = 0$; namely, no firms sell assets in both periods. There are two possible cases: $\hat{\theta}_g = 0$ or $\hat{\theta}_g > 0$.

Suppose first $\hat{\theta}_g = 0$, there will be no asset trading in $t = 1$. Then, there is no updating on firms' types in $t = 2$, so $t = 2$ buyers hold the prior belief about firms' types, leading to $\theta_2 = \theta_0 > 0$. However, by definition of θ_0 and p_0 , a buyer in $t = 1$ can profitably deviate by offering $p' \in (I, p_0)$. Hence, $\hat{\theta}_g > 0$.

Next, suppose $\hat{\theta}_g > 0$. Once again, there are two possible cases, $\hat{\theta}_g = \theta_2$ and $\hat{\theta}_g < \theta_2$. We first show that it is impossible to have $0 < \hat{\theta}_g = \theta_2 < 1$. If $0 < \hat{\theta}_g = \theta_2 < 1$, this means that no firms with $\theta > \hat{\theta}_g$ sell in $t = 2$. However, a buyer in $t = 1$ can profitably deviate by offering a $p' > \hat{\theta}_g - S$: the strict log-concavity property of $f(\cdot)$ implies $f(\cdot)/(1 - F(\hat{\theta}_g))$ is also strictly log-concave, and thus there exists a $\theta' > \hat{\theta}_g$ such that $p' = \theta' - S$ and $\mathbb{E}[\theta | \hat{\theta}_g < \theta \leq \theta'] - p' > 0$. If $0 < \hat{\theta}_g = \theta_2 = 1$, we have $\hat{\theta}_g > \theta_0$. By definition of θ_0 , however, a buyer in $t = 2$ can profitably deviate by offering $p' = p_0 - \varepsilon$ for a sufficiently small ε to the bailout recipients.

Consider next $0 < \hat{\theta}_g < \theta_2 \leq 1$. Since a firm sells only in $t = 1$ to the government at p_g or only in $t = 2$ to the market at p_2^m (recall by hypothesis no firm sells in both periods), no arbitrage implies $p_g = p_2^m$. There are two possible cases: $\hat{\theta}_g \geq \theta_0$ or $\hat{\theta}_g < \theta_0$. If $\hat{\theta}_g \geq \theta_0$, a buyer

in $t = 2$ can profitably deviate by offering $p' \in [I, p_0)$ to the firms that sold to the government in $t = 1$. If $\hat{\theta}_g < \theta_0$, a buyer in $t = 1$ can profitably deviate by offering $p' \in (\max\{\hat{\theta}_g - S, I\}, p_0)$: since buyers in $t = 2$ will offer $p_2^m = p_g$ to any firm that did not sell to the government, all firms with types $\theta \leq p' + S$ will sell at p' in $t = 1$ and the sale is profitable for the buyer (by definition of p_0). Consequently, there cannot exist any equilibrium with $\hat{\theta} = 0$.

Step 2. $\hat{\theta} < \max\{\hat{\theta}_g, \theta_2\}$.

Suppose to the contrary that there is an equilibrium with $0 < \hat{\theta} = \hat{\theta}_g = \theta_2$. This means that no firm sells only in one period. We first show that there are positive measures of firms sell to the government and to the market in $t = 1$ (that is, in terms of our notation in the text, $\mu_g \in (0, 1)$). If all types $\theta \leq \hat{\theta}$ refuse the bailout, the equilibrium is same as that without bailouts, and thus $\hat{\theta} = \theta_0$. Since $p_g > p_0$, however, types $\theta \in (\theta_0, p_g + S]$ will deviate and sell to the government in $t = 1$. If all types $\theta \leq \hat{\theta}$ accept the bailout, there are two possible cases, either $\hat{\theta} > \theta_0$ or $\hat{\theta} \leq \theta_0$. In the former case, we must have $\hat{\theta} \leq p_2^g + S$, which implies $p_2^g > p_0$. However, such a price cannot break even for the $t = 2$ buyers. In the latter case, firms with types $\theta \in (\hat{\theta}, p_g + S]$ will deviate and sell to the government in $t = 1$. We have thus proven that positive measures of firms sell to the government and to the market in $t = 1$.

Since all firms that sold to the market in $t = 1$ also sell in $t = 2$, the zero-profit condition and the supposition $\hat{\theta} = \hat{\theta}_g = \theta_2$ imply $p_m = p_2^m$. For expositional convenience, we throughout abuse the notation p_m as the equilibrium price offered in both periods to the firms that did not sell to the government. Since a type- $\hat{\theta}$ firm is indifferent between selling in both periods and not selling in any period, we must have $2\hat{\theta} = 2p_m + 2S$, or equivalently $\hat{\theta} = p_m + S$. If $p_m < p_g$, types $\theta \in (\hat{\theta}, p_g + S]$ will deviate and sell to the government in $t = 1$. Hence, we must have $p_m \geq p_g$, and thus $\hat{\theta} \geq p_g + S > \theta_0$. Since the no-arbitrage condition for types $\theta \leq \hat{\theta}$ implies $p_g + p_2^g = 2p_m$, we must have $p_m \leq p_2^g$. Since the equilibrium prices p_m and p_2^g must break even for the $t = 2$ buyers, we must have $\mathbb{E}[\theta | \theta \leq \hat{\theta}] = \mu_g p_2^g + (1 - \mu_g)p_m$, which implies

$$\mathbb{E}[\theta | \theta \leq \hat{\theta}] + S \geq p_m + S = \hat{\theta}.$$

By the definition of θ_0 (see (1)), we must then have $\hat{\theta} \leq \theta_0$, but this contradicts an earlier observation that $\hat{\theta} > \theta_0$. Therefore, an equilibrium with $0 < \hat{\theta} = \hat{\theta}_g = \theta_2$ cannot exist for any $p_g > p_0$. *Q.E.D.*

C Detailed Analysis of Equilibria

In the text, we characterized the necessary conditions for both types of equilibria (short-lived stimulation and delayed stimulation types). In this section, we present all the necessary conditions in detail and show them they are sufficient for equilibrium.

C.1 Short-lived Stimulation Equilibrium

In a short-lived stimulation equilibrium, firms with types $\theta \leq \hat{\theta}$ sell in both periods: a fraction μ_g of these firms sell to the government at p_g in $t = 1$ and to the market at p_2^g in $t = 2$, and the remaining fraction $\mu_m = 1 - \mu_g$ of them sell to the market at p_m in both periods. In addition, types $\theta \in (\hat{\theta}, \hat{\theta}_g]$ sell to the government at p_g in $t = 1$ but do not sell in $t = 2$ (i.e., boycott the $t = 2$ market), and types $\theta > \hat{\theta}_g$ do not sell in either period. Let $\bar{\theta}_g$ denote the average value of types of the bailout recipients that sell to the market in $t = 2$, and let $\bar{\theta}_m$ denote the average value of types that sell to the market in both periods. The zero-profit condition implies that $p_2^g = \bar{\theta}_g$ and $p_m = \bar{\theta}_m$.

There are several necessary conditions for the existence of short-lived stimulation equilibria. First, recall from the text the necessary conditions associated with the optimality of firms' prescribed equilibrium strategies:

$$p_g + \bar{\theta}_g = 2\bar{\theta}_m, \quad (4)$$

$$2\bar{\theta}_m + 2S = \hat{\theta} + p_g + S, \quad (5)$$

$$\hat{\theta}_g = (p_g + S) \wedge 1. \quad (6)$$

Second, there are feasibility conditions on the endogenous variables $(\bar{\theta}_g, \bar{\theta}_m, \mu_g, \hat{\theta})$. One of these conditions is that every price offered to firms in equilibrium must be sufficiently high to fund the cost of investment I . By Theorem 3, we have

$$p_2^g = \bar{\theta}_g < p_m = \bar{\theta}_m < p_g. \quad (18)$$

Thus, we must have

$$\bar{\theta}_g \geq I. \quad (19)$$

Furthermore, by definition of $\bar{\theta}_g, \bar{\theta}_m$, and μ_g , we must have

$$\mu_g \bar{\theta}_g + \mu_m \bar{\theta}_m = \mathbb{E}[\theta | \theta \leq \hat{\theta}], \quad (20)$$

which is same as (3) in the text. Lastly, feasibility requires that $\bar{\theta}_g$ cannot be too low: specifically, we must have

$$\bar{\theta}_g \geq \mathbb{E}[\theta | \theta \leq F^{-1}(\mu_g F(\hat{\theta}))]. \quad (21)$$

Lastly, there are two other necessary conditions. First, buyers in $t = 2$ should not have an incentive to offer a $p' \neq p_2^g = \bar{\theta}_g$ to the firms that received a bailout in $t = 1$. If a $t = 2$ buyer offers a $p' > \bar{\theta}_g$ to the bailout recipients, all bailout recipients with types $\theta \leq \hat{\theta}$ and

types $\theta \in (\hat{\theta}, (p' + S) \wedge \hat{\theta}_g]$ will sell at p' . By (6), no buyer will want to offer $p' > p_g$, so we can focus on $p' \leq p_g$. Since a fraction μ_g of firms with types $\theta \leq \hat{\theta}$ receives a bailout in $t = 1$ and since their average value is $\bar{\theta}_g$, for such a deviating offer to be unprofitable, we must have

$$\frac{\mu_g F(\hat{\theta}) \bar{\theta}_g + \int_{\hat{\theta}}^{p'+S} \theta f(\theta) d\theta}{\mu_g F(\hat{\theta}) + (F(p' + S) - F(\hat{\theta}))} - p' < 0 \quad \forall p' \in (\bar{\theta}_g, p_g]. \quad (22)$$

Next, buyers in $t = 2$ should not have an incentive to offer a $p' \neq p_m$ to the firms that did not sell to the government in $t = 1$. For instance, if a buyer offers $p' > p_g$ in $t = 2$, the firms that refused the bailout with types $\theta \leq \hat{\theta}$ and the additional firms with types $\theta \in (\hat{\theta}_g, (p' + S) \wedge 1]$ will sell their assets to the deviator. Again, since a fraction $1 - \mu_g$ of the firms with types $\theta \leq \hat{\theta}$ refused the bailout and their average value is $\bar{\theta}_m$, the necessary condition for such a p' not to be profitable is

$$\frac{(1 - \mu_g) F(\hat{\theta}) \bar{\theta}_m + \int_{\hat{\theta}_g}^{p'+S} \theta f(\theta) d\theta}{(1 - \mu_g) F(\hat{\theta}) + (F(p' + S) - F(\hat{\theta}_g))} - p' < 0 \quad \forall p' \in (p_g, 1 - S]. \quad (23)$$

The following proposition states that the necessary conditions listed above are indeed sufficient for the existence of the short-lived stimulation equilibrium.

Proposition 1. *For a $p_g > p_0$, there exists a short-lived stimulation equilibrium if there exist $(\mu_g, \bar{\theta}_g, \bar{\theta}_m, \hat{\theta})$ that satisfy (19) – (23) and (4) – (6).*

Proof. We throughout focus on the case $\hat{\theta}_g < 1$: the proof for the other case $\hat{\theta}_g = 1$ is very similar.

We first show that it is optimal for every type- θ firm to play the prescribed equilibrium strategies over two periods. Consider $t = 2$ first. After accepting a bailout, firms with types $\theta \leq \hat{\theta}$ find it optimal to sell to the market at $p_2^g = \bar{\theta}_g$ since $\theta \leq \hat{\theta} = \bar{\theta}_g + S$ from (6). The same condition implies that it is optimal for the firms with types $\theta \in (\hat{\theta}, \hat{\theta}_g]$ (after accepting the bailout in $t = 1$) not to sell at p_2^g . Consider now the firms with types $\theta \leq \hat{\theta}$ that sold to the market in $t = 1$. It is optimal for these firms to sell at $p_m = \bar{\theta}_m$ since $\theta \leq \hat{\theta} = \bar{\theta}_g + S < \bar{\theta}_m + S$ by Theorem 3-(i). Theorem 3-(i) also implies $\bar{\theta}_m + S < p_g + S = \hat{\theta}_g < \theta$, so it is optimal for the firms with types $\theta > \hat{\theta}_g$ not to sell at p_m .

Consider $t = 1$ next. From (4) and (5), firms with types $\theta \leq \hat{\theta}$ are indifferent between selling to the government in $t = 1$ and selling to the market in $t = 1$. Furthermore, (5) implies $\theta + p_g + S \leq 2p_m + 2S = p_g + p_2^g + 2S$ for all $\theta \leq \hat{\theta}$, so it is optimal for all firms with types $\theta \leq \hat{\theta}$ to play the prescribed equilibrium strategy. The same condition also implies $p_g + S + \theta > 2p_m + 2S$ for all $\theta > \hat{\theta}$. Moreover, (6) implies $2\theta > p_g + S + \theta$ if and only if $\theta > \hat{\theta}_g$. Hence, it is optimal for a type- θ firm to sell to the government in $t = 1$ if $\theta \in (\hat{\theta}, \hat{\theta}_g]$ and for a type- θ firm not to sell in $t = 1$ if $\theta > \hat{\theta}_g$.

We next show that it is optimal for every buyer in $t = 1, 2$ to play the prescribed equilibrium strategy, given all the other buyers play the same strategy. Note from the zero-profit condition that every equilibrium price will break even for buyers on the equilibrium path.

Consider buyers in $t = 2$ who make offers to the bailout recipients. Since all the other buyers offer $p_2^g = \bar{\theta}_g$, any offer $p' < p_2^g$ will be rejected and thus will not be profitable. Suppose a $t = 2$ buyer offers $p' > \bar{\theta}_g$. This offer will attract all of the bailout recipients with types $\theta \leq \hat{\theta}$ and those with types $\theta \in (\hat{\theta}, (p' + S) \wedge 1]$. Thus, the deviating buyer will get the payoff

$$\frac{\mu_g F(\hat{\theta}) \bar{\theta}_g + \int_{\hat{\theta}}^{(p'+S) \wedge 1} \theta f(\theta) d\theta}{\mu_g F(\hat{\theta}) + (F((p' + S) \wedge 1) - F(\hat{\theta}))} - p'.$$

However, the payoff above is negative by (22), and therefore, no $p' > \bar{\theta}_g$ is profitable. Therefore, it is optimal for a $t = 2$ buyer to offer $p_2^g = \bar{\theta}_g$ to the bailout recipients.

Consider next buyers in $t = 2$ who make offers to the firms that did not accept a bailout offer. By the same logic as above, one can easily find that it is not optimal for any buyer to offer $p' < p_m = \bar{\theta}_m$. Furthermore, it is not optimal for any buyer to offer $p' \in (\bar{\theta}_m, p_g]$ since such an offer will attract the sellers to the $t=1$ market, whose average value is $\bar{\theta}_m$, so the buyer will earn $\bar{\theta}_m - p' < 0$. Lastly, suppose a $t = 2$ buyer offers $p' > p_g$. Such an offer will attract all firms that sold to the market in $t = 1$ plus all types $\theta \in (\hat{\theta}_g, (p' + S) \wedge 1]$. The resulting deviation payoff is

$$\frac{(1 - \mu_g) F(\hat{\theta}) \bar{\theta}_m + \int_{\hat{\theta}_g}^{(p'+S) \wedge 1} \theta f(\theta) d\theta}{(1 - \mu_g) F(\hat{\theta}) + (F((p' + S) \wedge 1) - F(\hat{\theta}_g))} - p'.$$

By (23), however, the above payoff is negative, which implies such a deviation price $p' > p_g$ is unprofitable. Hence, it is optimal for every $t = 2$ buyer to offer $p_m = \bar{\theta}_m$ to the firms refusing the bailout.

Lastly, consider buyers in $t = 1$. We shall prove that it is optimal for a buyer of $t = 1$ to offer p_m . To this end, we first establish a property on $\bar{\theta}_m$.

Lemma 4. $\bar{\theta}_m \geq p_0$ in any short-lived stimulation equilibrium.

Proof. Suppose to the contrary that $\bar{\theta}_m < p_0 = \mathbb{E}[\theta | \theta \leq \theta_0]$. Since $\bar{\theta}_g < \bar{\theta}_m$ by Theorem 3, we observe $\bar{\theta}_g < p_0$. Now, suppose a $t = 2$ buyer deviates and offers p_0 to the firms that accepted a bailout. These firms comprise a fraction μ_g of firms with $\theta \leq \hat{\theta}$ (whose average is $\bar{\theta}_g$) and all firms with $\theta \in (\hat{\theta}, \hat{\theta}_g]$. All of the former firms will sell to the deviator, and among the latter, all firms with $\theta \in (\hat{\theta}, \theta_0]$ are now willing to sell to the deviator at p_0 . (Recall $\hat{\theta}_g = p_g + S > p_0 + S = \theta_0$.) Thus the deviating buyer will get the payoff

$$\frac{\mu_g F(\hat{\theta}) \bar{\theta}_g + \int_{\hat{\theta}}^{\theta_0} \theta f(\theta) d\theta}{\mu_g F(\hat{\theta}) + F(\theta_0) - F(\hat{\theta})} - p_0.$$

Since $\mu_g \bar{\theta}_g + (1 - \mu_g) \bar{\theta}_m = \mathbb{E}[\theta | \theta \leq \hat{\theta}]$ by (20) and $\bar{\theta}_g < \bar{\theta}_m$, however, we have

$$\frac{\mu_g F(\hat{\theta}) \bar{\theta}_g + \int_{\hat{\theta}}^{\theta_0} \theta f(\theta) d\theta}{\mu_g F(\hat{\theta}) + F(\theta_0) - F(\hat{\theta})} = \frac{F(\theta_0) \mathbb{E}[\theta | \theta \leq \theta_0] - (1 - \mu_g) F(\hat{\theta}) \bar{\theta}_m}{F(\theta_0) - (1 - \mu_g) F(\hat{\theta})} > \mathbb{E}[\theta | \theta \leq \theta_0] = p_0.$$

This implies the deviation will be profitable, a contradiction. *Q.E.D.*

We now show that it is optimal for every $t = 1$ buyer to offer p_m . Since buyers in $t = 2$ will offer p_m to every firm refusing the bailout, any offer $p' < p_m = \bar{\theta}_m$ is not attractive to any firm. Suppose a buyer offers $p' > \bar{\theta}_m$. Since $p' + p_m + 2S > 2p_m + 2S = p_g + p_2^g + 2S = \hat{\theta} + p_g + S$, all firms with type $\theta \leq p' + S - (p_g - \bar{\theta}_m)$ will sell at p' . This implies the deviating buyer will get the payoff $\mathbb{E}[\theta | \theta \leq p' + S - (p_g - \bar{\theta}_m)] - p'$. Since $p' > p_m = \bar{\theta}_m \geq p_0$ by Lemma 4 and $p_g > \bar{\theta}_m$ by Theorem 3, however, we have

$$\mathbb{E}[\theta | \theta \leq p' + S - (p_g - \bar{\theta}_m)] - p' < \mathbb{E}[\theta | \theta \leq p' + S] - p' < 0,$$

where the strict inequality follows from definition of θ_0 . This implies any offer $p' > p_m$ is not profitable. *Q.E.D.*

C.2 Delayed-Stimulation Equilibrium

In a delayed stimulation equilibrium, firms with types $\theta \leq \hat{\theta}$ sell in both periods: a fraction μ_g of these firms sell to the government at p_g in $t = 1$ and to the market at p_2^g in $t = 2$ and the rest of them sell to the market at p_m in $t = 1$ and p_2^m in $t = 2$. Meanwhile, types $\theta \in (\hat{\theta}, \hat{\theta}_g]$ sell to the government at p_g in $t = 1$ but withdraw from the $t = 2$ market, and types $\theta \in (\hat{\theta}_g, \theta_2]$ sell at p_2^m only in $t = 2$; types $\theta > \theta_2$ do not sell in any period. Recall from Theorem 4 that $\hat{\theta} = \theta_0$, $p_2^g = \bar{\theta}_g = p_m = \bar{\theta}_m = p_0 = \mathbb{E}[\theta | \theta \leq \theta_0]$, and $p_2^m = p_g$ in any delayed-stimulation equilibrium.

Throughout, we focus on delayed stimulation equilibria in which $0 < \mu_g < 1$ and $\hat{\theta} < \hat{\theta}_g$ —one can easily establish an equilibrium without this property by applying the same logic. Like we did in Section C.1, we first list all necessary conditions for a delayed stimulation equilibrium. First, like in the short-lived stimulation equilibrium, there are conditions associated with the optimality of firms' prescribed equilibrium strategies as listed below:

$$p_g + p_2^g = p_m + p_2^m, \tag{24}$$

$$p_m + p_2^m + 2S = \hat{\theta} + p_g + S, \tag{25}$$

$$\theta_2 = (p_2^m + S) \wedge 1. \tag{26}$$

By applying $p_2^g = p_m = p_0$ to the conditions above, we have

$$p_g = p_2^m, \quad (27)$$

$$\hat{\theta} = p_0 + S = \theta_0, \quad (28)$$

$$\theta_2 = (p_g + S) \wedge 1. \quad (29)$$

Second, buyers must find it optimal to offer the equilibrium price on the equilibrium path over two periods. First of all, buyers in $t = 2$ should obtain the highest payoff from offering p_0 to the firms that received a bailout in $t = 1$. Specifically, the $t = 2$ buyers must not have an incentive to offer $p' > p_0$ to the bailout recipients. By definition of θ_0 , such a p' will attract the bailout recipients with types $\theta \leq \theta_0$ and $\theta \in (\theta_0, p' + S]$. Since the fraction μ_g of firms with types $\theta \leq \theta_0$ receive a bailout in $t = 1$ and their average value is $\bar{\theta}_g = \mathbb{E}[\theta | \theta \leq \theta_0]$, the following condition must hold for p' to be unprofitable:

$$\frac{\mu_g F(\theta_0) \mathbb{E}[\theta | \theta \leq \theta_0] + \int_{\theta_0}^{p'+S} \theta f(\theta) d\theta}{\mu_g F(\theta_0) + (F(p' + S) - F(\theta_0))} - p' < 0 \quad \forall p' \in (p_0, p_g]. \quad (30)$$

Moreover, buyers in $t = 2$ must get the highest payoff from offering p_0 to the firms that refused a bailout offer in $t = 1$. As before, this means that a $t = 2$ buyer must not have an incentive to offer $p' > p_2^m = p_g$ to these bailout holdouts. Since the fraction $1 - \mu_g$ of firms with types $\theta \leq \theta_0$ do not receive the bailout in $t = 1$ and their average value is $\mathbb{E}[\theta | \theta \leq \theta_0]$, we must have the following condition:

$$\frac{(1 - \mu_g) F(\theta_0) \mathbb{E}[\theta | \theta \leq \theta_0] + \int_{\hat{\theta}_g}^{p'+S} \theta f(\theta) d\theta}{(1 - \mu_g) F(\hat{\theta}) + (F(p' + S) - F(\theta_0))} - p' < 0 \quad \forall p' > p_g. \quad (31)$$

Indeed, the following observation reveals that the above necessary conditions—as well as the properties of Theorem 4—are sufficient for the existence of the delayed stimulation equilibrium.

Proposition 2. *For a $p_g > p_0$, there exists a delayed stimulation equilibrium if there exist $\mu_g \in (0, 1]$ and $\hat{\theta}_g \in [\theta_0, p_g + S]$ that satisfy (24) – (31).*

Proof. Just as we did in the proof of Proposition 1, we focus on the case $\theta_2 < 1$; the proof for the case $\theta_2 = 1$ is very similar.

We first prove that it is optimal for every type- θ firm to play the prescribed equilibrium strategies over two periods. Consider $t = 2$ first. By definition of θ_0 and p_0 , one can easily show that after receiving the bailout in $t = 1$, firms with types $\theta \leq \theta_0$ would sell whereas firms with types $\theta \in (\theta_0, \hat{\theta}_g]$ would refuse to do so in $t = 2$. Furthermore, since $\theta_2 = p_g + S$ by Theorem 4, it is optimal for every firm with type $\theta \leq \theta_2$ to sell at $p_2^m = p_g$ in $t = 2$ after refusing the bailout in $t = 1$. Lastly, since $\theta_2 = p_g + S$ and $p_2^m = p_g$, it is optimal for the firms

with types $\theta > \theta_2$ not to sell at p_2^m in $t = 2$.

Consider $t = 1$ next. Since $p_g + p_2^g = p_m + p_2^m$ from (27), firms with types $\theta \leq \theta_0$ are indifferent between selling to the government and selling to the market in $t = 1$. Furthermore, since $\theta + p_g + S \leq p_g + p_0 + 2S$ for all $\theta \leq \theta_0$ from (28), it is optimal for the firms with types $\theta \leq \theta_0$ to sell in $t = 1$. The same condition (28) also implies $p_g + p_0 + 2S < \theta + p_g + S$ for all $\theta > \theta_0$, so firms with types $\theta > \theta_0$ do not prefer selling in $t = 1$. Furthermore, since $p_2^m = p_g > p_0$, firms with types $\theta \in (\theta_0, \theta_2]$ are indifferent between accepting the bailout in $t = 1$ (but boycotting the market in $t = 2$) and not selling in $t = 1$ (but selling in $t = 2$). Moreover, since $\theta_2 = p_g + S$, it is optimal for the firms with types $\theta \in (\theta_0, \hat{\theta}_g]$ to sell to the government in $t = 1$ and for the firms with types $\theta > \hat{\theta}_g$ not to sell in $t = 1$.

Next, we prove that it is optimal for buyers to offer the stated equilibrium prices in each period. First, consider the offers made to the firms that received a bailout in $t = 1$. In $t = 1$, a fraction μ_g of firms with types $\theta \leq \theta_0$ and all firms with types $\theta \in (\theta_0, \hat{\theta}_g]$ receive the bailout. Since any offer $p' < p_2^g = p_0$ is unattractive to these firms, no $t = 2$ buyer will deviate and offer $p' < p_0$. Suppose a $t = 2$ buyer offers $p' > p_0$. Such an offer will attract the bailout recipients with types $\theta \leq \theta_0$ and $\theta \in (\theta_0, p' + S]$. Since the average value of firms with types $\theta \leq \theta_0$ that sell to the government is $\bar{\theta}_g = \mathbb{E}[\theta | \theta \leq \theta_0]$, the deviating buyer will get the payoff

$$\frac{\mu_g F(\theta_0) \mathbb{E}[\theta | \theta \leq \theta_0] + \int_{\theta_0}^{p'+S} \theta f(\theta) d\theta}{\mu_g F(\theta_0) + (F(p' + S) - F(\theta_0))} - p'.$$

However, such an offer p' is not profitable by (30): if $p' \in (p_0, p_g]$, the payoff from offering p' is negative; the payoff is strictly negative for all $p' > p_g$.

Consider next offers made to the firms that refused a bailout in $t = 1$. In $t = 1$, a fraction $(1 - \mu_g)$ of firms with types $\theta \leq \theta_0$ refuse the bailout. Furthermore, the firms with higher types $\theta > \hat{\theta}_g$ refuse the bailout in $t = 1$. Since any offer $p' < p_0$ is unattractive to the firms refusing the bailout in $t = 1$, no $t = 2$ buyer will offer such a p' . Suppose a $t = 2$ buyer deviates and offers $p' > p_0$. If $p' \leq \hat{\theta}_g - S$, only the firms with types $\theta \leq \theta_0$ will sell at p' , making p' unprofitable since $\bar{\theta}_m - p' = \mathbb{E}[\theta | \theta \leq \theta_0] - p' < 0$. If $p' > \hat{\theta}_g - S$, the firms that sold to the market in $t = 1$ and the holdout firms with types $\theta \in (\hat{\theta}_g, (p' + S) \wedge 1]$ sell to the deviating buyer. Since a fraction $(1 - \mu_g)$ of firms with types $\theta \leq \theta_0$ sell to the market in $t = 1$ and their average value is $\mathbb{E}[\theta | \theta \leq \theta_0]$, the deviating buyer will get the payoff

$$\frac{(1 - \mu_g) F(\theta_0) \mathbb{E}[\theta | \theta \leq \theta_0] + \int_{\hat{\theta}_g}^{(p'+S) \wedge 1} \theta f(\theta) d\theta}{(1 - \mu_g) F(\hat{\theta}) + (F((p' + S) \wedge 1) - F(\theta_0))} - p'.$$

However, the above payoff is negative by (31).

Lastly, consider buyers in $t = 1$. If a buyer offers $p' \neq p_0$ and a type- θ firm sells at that

price, this firm will receive the price offer $p_2^m = p_g$ from buyers in $t = 2$: recall the $t = 2$ buyers can only observe whether or not firms receive the bailout. Since $p' + p_g + 2S < p_0 + p_g + 2S$ for any $p' < p_0$, any offer $p' < p_0$ is unattractive to any type- θ firm, and thus no buyer in $t = 1$ has an incentive to offer such a price. Suppose a buyer in $t = 1$ deviates and offers $p' > p_0$. Since $p' + p_g + 2S > p_0 + p_g + 2S$, all firms with types $\theta \leq \theta_0$ will also sell at p' . Furthermore, since $p' + p_g + 2S > \theta_0 + p_g + S$, firms with types $\theta \in (\theta_0, (p' + S) \wedge 1]$ will also sell at p' . This implies the deviating buyer will get the payoff $\mathbb{E}[\theta | \theta \leq (p' + S) \wedge 1] - p'$. However, by definition of θ_0 and p_0 , we have $\mathbb{E}[\theta | \theta \leq (p' + S) \wedge 1] - p' < 0$ for all $p' > p_0$, which implies any offer $p' > p_0$ is unprofitable. Q.E.D.

We next check the existence of delayed stimulation equilibria. To this end, we first define a function $\gamma : [0, 1] \rightarrow [0, 1]$ as

$$\gamma(a) := \max\{x > a : x - S \leq \mathbb{E}[\theta | \theta \in [a, x]]\}. \quad (32)$$

Log-concavity of $f(\cdot)$ guarantees that $\gamma(\theta)$ is well defined for every $\theta \in [0, 1]$. Also note that $\gamma(\theta)$ is a continuous and increasing function of θ . The following observation reveals that any bailout offer p_g always induces a delayed stimulation equilibrium if p_g is sufficiently high.

Proposition 3. *There exists a delayed-stimulation equilibrium if $p_g \geq \mathbb{E}[\theta | \theta_0 \leq \theta \leq \gamma(\theta_0)]$.*

Proof. Fix any $p_g \geq \mathbb{E}[\theta | \theta_0 \leq \theta \leq \gamma(\theta_0)]$ and take $\hat{\theta}_g$ as a unique solution to $p_g = \mathbb{E}[\theta | \theta \in (\hat{\theta}_g, \gamma(\hat{\theta}_g))]$. Note from the definition of $\gamma(\cdot)$ as above that $\gamma(\hat{\theta}_g) = (p_g + S) \wedge 1$. In what follows, we will prove that there is a delayed stimulation equilibrium in which $\mu_g = 1$, $p_g = \mathbb{E}[\theta | \theta \in (\hat{\theta}_g, \gamma(\hat{\theta}_g))]$, and $\theta_2 = \gamma(\hat{\theta}_g)$.

We first show that it is optimal for every type θ firm to play the prescribed equilibrium strategies over two periods. Consider $t = 2$ first. By definition of θ_0 , p_0 , and $\gamma(\hat{\theta}_g)$, it is optimal for a type θ firm to sell at price p_0 in $t = 2$ if $\theta \leq \theta_0$ and sell at $p_2 = p_g$ if $\theta \in (\hat{\theta}_g, \theta_2]$, whereas firms with types $\theta \in (\theta_0, \hat{\theta}_g]$ hold out rather than sell at p_0 (the price the firms are offered) in $t = 2$ and firms with types $\theta > \theta_2$ hold out rather than sell at $p_2 = p_g$. Consider $t = 1$ next. Since $\mu_m = 0$, there is no market offer in $t = 1$. Furthermore, if a firm sells to the government in $t = 1$, it will receive the market offer p_0 in $t = 2$. If a firm does not sell in $t = 1$, it will receive the market offer $p_2 = p_g$ in $t = 2$. Since $p_0 + p_g + 2S \geq \theta + p_g + S$ if and only if $\theta \leq \theta_0$, it is optimal for firms with types $\theta \leq \theta_0$ to sell to the government in $t = 1$. If firms with types $\theta \in (\theta_0, \theta_2]$ sell to the government in $t = 1$, these firms will get the total payoff $\theta + p_g + S$ since they will not sell at p_0 in $t = 2$. If firms with the same types do not sell in $t = 1$, their total payoff is $\theta + p_g + S$ since they will sell at $p_2 = p_g$ to the market in $t = 2$. Once again, since $p_0 + p_g + 2S < \theta + p_g + S$ if and only if $\theta > \theta_0$, it is weakly optimal for the firms with types $\theta \in (\theta_0, \hat{\theta}_g]$ to sell to the government and for the firms with types $\theta \in (\hat{\theta}_g, \theta_2]$

not to sell in $t = 1$. Lastly, since $2\theta > \theta + p_g + S$ if and only if $\theta > p_g + S$, it is optimal for firms with types $\theta > \theta_2$ not to sell in any period.

We next show that it is optimal for buyers to offer the equilibrium prices in each period. Consider $t = 2$ first. Given the equilibrium behavior of the firms, those firms that sell to the government in $t = 1$ are believed to be of types $\theta \leq \hat{\theta}_g$; and those that do not sell to the government in $t = 1$ are believed to be of types $\theta > \hat{\theta}_g$. Since $\hat{\theta}_g \geq \theta_0$, it is optimal for the buyers to offer $p_0 = \mathbb{E}[\theta | \theta \leq \theta_0]$ to the firms that sold to the government in $t = 1$ ³¹ and offer $p_2 = \mathbb{E}[\theta | \theta \in [\hat{\theta}_g, \gamma(\hat{\theta}_g)]]$ to the firms that did not sell. Consider $t = 1$ next. Suppose a buyer deviates and offers $p' \geq I$ to the firms. Note that if a firm accepts this offer, then it will receive the market offer $p_2 = p_g$ in $t = 2$. If $p' \leq p_0$, no firm will sell at p' since $p' + p_2 + 2S \leq p_g + p_0 + 2S$. Suppose $p' > p_0$, then every type- θ firm will sell at p' if and only if

$$p' + p_g + 2S \geq \theta + p_2 + S \iff \theta \leq p' + S.$$

Since $p' > p_0$, however, such an offer is not profitable for the deviator (i.e., $\mathbb{E}[\theta | \theta \leq p' + S] - p' < 0$). In sum, it is optimal for any $t = 1$ buyer not to make any offer. *Q.E.D.*

³¹Recall p_0 is the highest profitable price for the buyers.