Caps on Political Lobbying: Reply

By Yeon-Koo Che and Ian L. Gale*

Yeon-Koo Che and Ian L. Gale (1998) (CG, hereafter) studied the impact of imposing a cap on lobbying expenditures. They showed that a cap may lead to (a) greater expected aggregate expenditure, and (b) a less efficient allocation of a political prize. In their comment, Todd Kaplan and David Wettstein (2006) (KW, hereafter) show that if the cap is not *rigid* (i.e., its impact on the cost of lobbying is continuous) it has no effect.

KW employ the same basic framework as CG, except for the assumption that a bid of x costs a lobbyist $c(x)$ for a strictly increasing, continuous function, $c(\cdot)$. Imposition of a cap raises costs to the strictly increasing, continuous function, $\bar{c}(\cdot)$.[1] To see why the cap has no effect on lobbying expenditures in that setting, think of a lobbyist choosing a *cost,* $\tilde{c} \in [0, \infty)$, rather than a bid. The lobbyist who chooses the higher cost necessarily makes the higher bid because the lobbyists have the same strictly increasing cost function. The functional relationship between bids and costs does not matter, so the cap has no effect.

We explore the reasons for the different results and we show that CG's results can still hold in a more general environment. Two components underlie CG's analysis: (a) a cap will constrain the stronger lobbyist, thereby leveling the playing field; and (b) this will intensify competition, raising the expected aggregate expenditure. In the case of KW's nonrigid cap, the first effect does not arise since the stronger lobbyist can always outspend the weaker one.[2] This does not vitiate the second component, however. As will be seen, when the cap has an equalizing effect, it will intensify competition, with the predicted effect on expenditures.

Below we characterize the precise nature of an "equalizing shift" in costs. More important, we will describe plausible circumstances under which a nonrigid cap can generate an equalizing shift when lobbyists differ in their costs of lobbying. (For instance, one lobbyist could be a more effective fundraiser than the other.) In such a case, a cap on lobbying reduces the competitive gap between the lobbyists, and it may cause the expected aggregate expenditure and the probability of misallocation to rise, just as in CG.

I. Model with Asymmetric Lobbying Costs

Following KW and CG, we model lobbying as an all-pay auction, so the high bid wins and all bids are forfeited. (We therefore refer to "lobbyist *i*" as "bidder *i*.") The environment here is more general, however, since we allow for differences in the cost of bidding. Bidder $i = 1, 2$ values the prize at v_i, and incurs the cost $c_i(x)$, when she bids $x \geq 0$.[3] We assume that $v_1 \geq v_2$ and $c_1(\cdot) \leq c_2(\cdot)$. We also assume that $c_i(\cdot)$ is continuous, strictly increasing, and unbounded, with $c_i(0) = 0$. Let $\mathcal{C}$ denote the set of cost function pairs satisfying the properties above. Finally, let $\mathcal{C}^* \subset \mathcal{C}$ denote the set of pairs that also satisfy the plausible condition $c_1'(\cdot) < c_2'(\cdot)$.

We will show how a cap may constrain the strong bidder more than the weak bidder, and how this change may again raise aggregate spending. We first provide the equilibrium characterization given asymmetric cost functions and the implications for the expected aggregate cost.[4]

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[1] The main difference in CG was the discontinuous effect of the cap. CG assumed $c(x) = x$ for all x. When there was a cap equal to $\bar{m}$, costs became $\bar{c}(x) = x$ for $x \leq \bar{m}$ and $\bar{c}(x) = \infty$ for $x > \bar{m}$. This meant that choosing costs above $\bar{m}$ was not an option.

[2] Recall that this is precisely what was not possible in CG. In that model, when the weaker lobbyist bids the cap, the stronger lobbyist cannot outspend him.

[3] In standard all-pay auctions, only the two strongest bidders are active if they have strictly higher valuations than the rest (see Michael R. Baye et al., 1996). The analogous result holds here, so there is little loss in considering only two bidders.

[4] We will henceforth refer to the expected aggregate "cost" rather than "expenditure," since lobbying costs may take forms besides monetary expenditures.

The highest bid that bidder 2 could profitably make is $\bar{x} := c_2^{-1}(v_2)$; it would give a payoff of $v_2 - c_2(\bar{x}) = 0$ if it were to win. Proceeding as in KW, we can show that the equilibrium support is $[0, \bar{x}]$. Bidders 1 and 2 receive equilibrium expected payoffs equal to $v_1 - c_1(\bar{x}) \geq 0$ and 0, respectively.

Let $F_i(\cdot)$ denote the cdf of bidder i's equilibrium bids. Bidder 1's expected payoff from a bid of x is

$$(1) \qquad v_1 F_2(x) - c_1(x) = v_1 - c_1(\bar{x}), \quad \forall x \leq \bar{x};$$

and bidder 2's is

$$(2) \qquad v_2 F_1(x) - c_2(x) = 0, \qquad \forall x \leq \bar{x}.$$

The equilibrium bid distributions are then

$$(3) \qquad F_1(x) = \frac{c_2(x)}{v_2} \qquad \text{and}$$

$$F_2(x) = \frac{v_1 - c_1(\bar{x}) + c_1(x)}{v_1}, \qquad \forall x \leq \bar{x}.$$

The equilibrium characterization follows.

LEMMA 1: *Given* $(c_1(\cdot), c_2(\cdot)) \in C$, *the unique equilibrium has the bidders bidding according to the cdfs in* (3). *The expected aggregate cost is*

$$(4) \qquad \left(\frac{v_2}{v_1}\right) c_1(c_2^{-1}(v_2)) + \left[\frac{1}{v_2} - \frac{1}{v_1}\right] \int_0^{v_2} c_1(c_2^{-1}(a))\, da.$$

PROOF:

That the described behavior constitutes an equilibrium follows directly from the construction of the cdfs. Uniqueness follows from standard arguments (see Baye et al., 1996).

The cdfs in (3) can be used to calculate the expected aggregate cost:

$$(5) \qquad \int_0^{\bar{x}} c_1(x)\, dF_1(x) + \int_0^{\bar{x}} c_2(x)\, dF_2(x)$$

$$= \int_0^{\bar{x}} \frac{c_1(x)c_2'(x)}{v_2}\, dx + \int_0^{\bar{x}} \frac{c_1'(x)c_2(x)}{v_1}\, dx$$

$$= \int_0^{\bar{x}} \frac{c_1(x)c_2'(x) + c_1'(x)c_2(x)}{v_1}\, dx + \left[\frac{1}{v_2} - \frac{1}{v_1}\right] \int_0^{\bar{x}} c_1(x)c_2'(x)\, dx$$

$$= \frac{c_1(\bar{x})c_2(\bar{x})}{v_1} + \left[\frac{1}{v_2} - \frac{1}{v_1}\right] \int_0^{\bar{x}} c_1(x)c_2'(x)\, dx$$

$$= \left(\frac{v_2}{v_1}\right) c_1(c_2^{-1}(v_2)) + \left[\frac{1}{v_2} - \frac{1}{v_1}\right] \int_0^{v_2} c_1(c_2^{-1}(a))\, da.$$

The first equality follows from (3), the second from adding and subtracting the same expression, the third from integration, and the last from $c_2(\bar{x}) = v_2$ and the change of variables $a := c_2(x)$.

We next present a general result concerning the effect of a reduction in the competitive gap between bidders. First, however, we define a change in the cost structure that reduces the gap:

DEFINITION 1: *Given any two cost structures,* $(c_1(\cdot), c_2(\cdot))$ *and* $(\bar{c}_1(\cdot), \bar{c}_2(\cdot))$ *in* C, *a shift from the former to the latter is an* equalizing shift *if*

$$(6) \qquad \bar{c}_1(\bar{c}_2^{-1}(a)) \geq c_1(c_2^{-1}(a)), \ \forall a \leq v_2.$$

It is a strictly equalizing shift *if the inequality holds strictly at* $a = v_2$.[5]

[5] In actuality, there is a strictly equalizing shift if the inequality holds strictly at $a = v_2$, or if $v_2 < v_1$ *and* the

An equalizing shift has a straightforward interpretation. Fix a bid, x, that might be made in equilibrium, given the initial cost functions. That bid would cost bidder 2 an amount $c_2(x)$. A bid that would cost her $c_2(x)$ after the shift would cost bidder 1 (weakly) more after the shift than before. That is, if x' satisfies $\bar{c}_2(x') = c_2(x)$, then $\bar{c}_1(x') \geq c_1(x)$. In the case where $(c_1(\cdot), c_2(\cdot)) \in C^*$, a sufficient condition for an equalizing shift is:

$$(7) \quad \bar{c}_1(x) - c_1(x) \geq \bar{c}_2(x) - c_2(x), \quad \forall x \leq c_2^{-1}(v_2).$$[6]

An equalizing shift arises in that case if bidder 1's cost function rises more than bidder 2's, for every bid, as depicted in Figure 1. Bidder 1 still has lower costs than bidder 2, but the advantage has fallen.

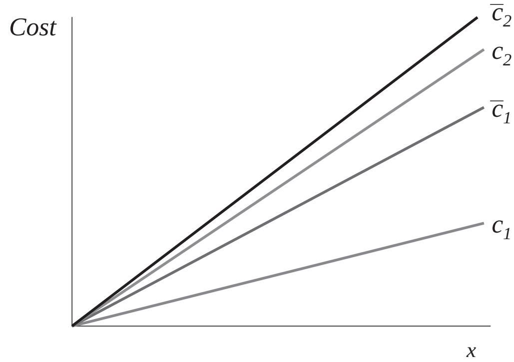

FIGURE 1. EQUALIZING SHIFT

PROPOSITION 1: *An equalizing shift in the cost structure raises the expected aggregate cost; a strictly equalizing shift raises it strictly.*

PROOF:

Using (4), the change in expected aggregate cost following a shift from $(c_1(\cdot), c_2(\cdot))$ to $(\bar{c}_1(\cdot), \bar{c}_2(\cdot))$ is:

$$(8) \quad \left(\frac{v_2}{v_1}\right)[\bar{c}_1(\bar{c}_2^{-1}(v_2)) - c_1(c_2^{-1}(v_2))] + \left[\frac{1}{v_2} - \frac{1}{v_1}\right] \int_0^{v_2} [\bar{c}_1(\bar{c}_2^{-1}(a)) - c_1(c_2^{-1}(a))]\, da.$$

Given an equalizing shift, both terms are nonnegative, which gives the first result. With a strictly equalizing shift, the first term is strictly positive, which gives the second result.

Proposition 1 generalizes a well-known result concerning the impact of asymmetry on rent dissipation to an environment in which costs are asymmetric.[7] Reducing the asymmetry generates more intense rivalry, leading to a higher expected aggregate cost. This will mean that if a bidding cap reduces the asymmetry, it will have that same effect.[8]

II. A Cap on Lobbying Expenditures

KW suggested two scenarios concerning nonrigid enforcement of a cap: (a) a cap is enforced imperfectly, or (b) bidders raise nonmonetary expenditures when facing a cap. We study both scenarios and show that the expected aggregate cost may rise in either one.

A. *Imperfect Enforcement Scenario*

Suppose that the bidders make purely monetary bids. When there is no cap, the bidders have cost functions in C. Now consider a cap

inequality holds for a positive measure of a in $[0, v_2]$. The former condition is simpler, and it is also necessary for the latter if the cost function pairs are in C^*. Hence, we focus on the former condition.

[6] To see this, suppose that the latter condition holds but the change is not an equalizing shift. Then, there exist a cost, $a \leq v_2$, and bids, $x' < x$, with $a = \bar{c}_2(x') = c_2(x)$, such that $\bar{c}_1(x') < c_1(x)$. This implies

$$\bar{c}_1(x') - \bar{c}_2(x') < c_1(x) - c_2(x) = c_1(x') - c_2(x') + \int_{x'}^{x} [c_1'(\tilde{x}) - c_2'(\tilde{x})]\, d\tilde{x} < c_1(x') - c_2(x'),$$

contradicting (7).

[7] When bidding costs satisfy $c_1(\cdot) = c_2(\cdot)$, the expected aggregate cost is $(v_2/2)[1 + (v_2/v_1)]$, which rises as the higher valuation, v_1, falls.

[8] In fact, Proposition 1 is also general enough to imply KW's finding: if $c_1(\cdot) = c_2(\cdot)$ and $\bar{c}_1(\cdot) = \bar{c}_2(\cdot)$, then a shift from either pair to the other is an equalizing shift, so the expenditure is unchanged.

of $\bar{m} > 0$, which is not enforced perfectly. Specifically, a bid of x is subject to a fine of $\alpha(x - \bar{m})$, with $\alpha(\cdot)$ a weakly increasing, continuous function that equals zero if $x - \bar{m} \leq 0$ and is strictly positive if $x - \bar{m} > 0$. The cap simply changes a bidder's cost function from $c_i(x)$ to $\bar{c}_i(x) := c_i(x + \alpha(x - \bar{m}))$. Proposition 1 then implies that KW's result generalizes to this case.

COROLLARY 1: *Imposing a binding cap, $\bar{m} < \bar{x} = c_2^{-1}(v_2)$, with a fine of $\alpha(x - \bar{m})$ for exceeding the cap, has no effect on the expected aggregate cost.*

PROOF:
Let

$$\phi(x) := x + \alpha(x - \bar{m}). \tag{9}$$

We then have

$$\begin{aligned} \bar{c}_1(\bar{c}_2^{-1}(a)) &= \bar{c}_1(\phi^{-1}(c_2^{-1}(a))) \\ &= c_1(\phi(\phi^{-1}(c_2^{-1}(a)))) \\ &= c_1(c_2^{-1}(a)), \ \forall a \leq v_2. \end{aligned} \tag{10}$$

The inverses are well-defined as the functions are all strictly increasing, and the first equality holds since $a = \bar{c}_2(x) = c_2(\phi(x))$ yields $x = \bar{c}_2^{-1}(a) = \phi^{-1}(c_2^{-1}(a))$. Equation (10) means that a shift from $(c_1(\cdot), c_2(\cdot))$ to $(\bar{c}_1(\cdot), \bar{c}_2(\cdot))$ is equalizing, as is a shift in the opposite direction. It follows from Proposition 1 that the expected aggregate cost must be equal under the two cost structures.

When bidders have asymmetric cost functions, imposing a fine has no effect on the expected aggregate cost. To see why, again think of the bidders choosing costs, $\tilde{c}_1$ and $\tilde{c}_2$. Bidder 1 wins if $c_1^{-1}(\tilde{c}_1) > c_2^{-1}(\tilde{c}_2)$. Now suppose that there is a cap. If the bidders select those same costs now, bidder 1 wins if $\phi^{-1}(c_1^{-1}(\tilde{c}_1)) > \phi^{-1}(c_2^{-1}(\tilde{c}_2))$, which is equivalent to $c_1^{-1}(\tilde{c}_1) > c_2^{-1}(\tilde{c}_2)$. Thus, the same bidder wins when there is a cap, for given costs. This means that bidders' incentives to incur lobbying costs are not affected by the cap.

We next consider penalties that do not directly increase the cost of a bid. Specifically, let a bid of x incur an expected penalty of $\beta(x - \bar{m})$, with $\beta(\cdot)$ a weakly increasing, continuous function that equals zero if $x - \bar{m} \leq 0$ and is strictly positive if $x - \bar{m} > 0$. The cap changes bidder i's cost function from $c_i(x)$ to $\bar{c}_i(x) := c_i(x) + \beta(x - \bar{m})$ here. This would arise if the penalty took a nonmonetary form such as incarceration, for example. We now show that the penalty makes a difference in this case.

COROLLARY 2: *Suppose that $(c_1(\cdot), c_2(\cdot)) \in C^*$. Imposing a binding cap, $\bar{m} < \bar{x} = c_2^{-1}(v_2)$, with a nonmonetary penalty of $\beta(x - \bar{m})$ for exceeding the cap, produces a strictly higher expected aggregate cost.*

PROOF:
By construction, $\bar{c}_2(\cdot) - \bar{c}_1(\cdot) = c_2(\cdot) - c_1(\cdot) \geq 0$, so $(\bar{c}_1(\cdot), \bar{c}_2(\cdot)) \in C$. Fix $a \in [0, v_2]$, and let $x' := \bar{c}_2^{-1}(a) \leq c_2^{-1}(a) =: x$. The latter inequality is strict if $a = v_2$ since $\beta(c_2^{-1}(a) - \bar{m}) \geq 0$, with a strict inequality if $a = v_2$. This yields

$$\begin{aligned} \bar{c}_1(\bar{c}_2^{-1}(a)) - c_1(c_2^{-1}(a)) &= \bar{c}_1(x') - c_1(x) \\ &= \bar{c}_1(x') - a - [c_1(x) - a] \\ &= \bar{c}_1(x') - \bar{c}_2(x') - [c_1(x) - c_2(x)] \\ &= c_1(x') - c_2(x') - [c_1(x) - c_2(x)] \\ &= \int_{x'}^{x} [c_2'(s) - c_1'(s)]\, ds \geq 0. \end{aligned} \tag{11}$$

The inequality holds since $x \geq x'$ and $c_2'(\cdot) > c_1'(\cdot)$; moreover, it is strict for $a = v_2$ since $x' < x$ in that case. Hence, the shift from $(c_1(\cdot), c_2(\cdot))$ to $(\bar{c}_1(\cdot), \bar{c}_2(\cdot))$ is a strictly equalizing shift. The result then follows from Proposition 1.

Imposition of the cap raises costs for both bidders, but the increase is relatively greater for bidder 1. The asymmetry between the bidders diminishes, which raises the expected aggregate cost.[9]

[9] Without a cap, bidder 1's equilibrium expected surplus was $v_1 - c_1(c_2^{-1}(v_2))$. When a fine was imposed, bidder 1's cost of making the supremum bid remained at $c_1(c_2^{-1}(v_2))$. With the nonmonetary penalty, by contrast, bidder 1's cost of making the supremum bid rose.

B. *Effort Diversion Scenario*

We now suppose that a bid comprises a monetary expenditure and a second component, which we call "effort." A cap on monetary expenditures may then induce bidders to substitute effort. Suppose that bidder $i = 1, 2$ incurs a cost of $\psi_i(m, e)$ when the monetary expenditure is m and effort is e. These two factors combine to produce a bid, $w(m, e)$. Assume that $\psi_1(m, e) \le \psi_2(m, e)$, $\forall(m, e) > (0, 0)$, so bidder 1 again has lower costs. In addition, let ψ_1, ψ_2 and w be continuous and strictly increasing in (m, e), and unbounded. Finally, assume that $\psi_i(0, 0) = w(0, 0) = 0$, ψ_i is quasi-convex, and w is quasi-concave. Bidder i's isocost curve (the locus of (m, e) with the same value of ψ_i) is then concave while the isobid curve (the locus of (m, e) giving the same value of w) is convex.

We now characterize the optimal composition of a bid. Given a cap on monetary expenditures, $\hat{m}$, bidder i's cost of bidding x is

$$(12) \quad \hat{c}_i(x; \hat{m}) := \min_{m,e}\{\psi_i(m, e) | w(m, e) = x \text{ and } m \le \hat{m}\}.$$

For $i = 1, 2$, let $c_i(\cdot) := \hat{c}_i(\,\cdot\,; \infty)$ denote the cost without a cap, and let $\bar{c}_i(\cdot) := \hat{c}_i(\,\cdot\,; \bar{m})$ denote the cost with a cap of $\bar{m}$.

Let $(m_i(x), e_i(x))$ denote bidder i's (interior) solution to the minimization problem in (12) when there is no cap. The solution occurs at the tangency of an isocost curve and the isobid curve, so it must satisfy

$$(13) \quad \frac{\partial \psi_i(m, e)/\partial m}{\partial \psi_i(m, e)/\partial e} = \frac{w_m(m, e)}{w_e(m, e)}.$$

Suppose that bidder 1 is relatively better at fundraising than bidder 2 is. Specifically, let

$$(14) \quad \frac{\partial \psi_1(m, e)/\partial m}{\partial \psi_1(m, e)/\partial e} < \frac{\partial \psi_2(m, e)/\partial m}{\partial \psi_2(m, e)/\partial e}, \quad \forall(m, e) \geqslant (0, 0).$$[10]

[10] An obvious example is $\psi_i(m, e) = \phi_i(m) + \xi(e)$, with $\phi_1'(\cdot) < \phi_2'(\cdot)$. Bidder 1 is more effective at fundraising than is bidder 2, but their effort costs are the same.

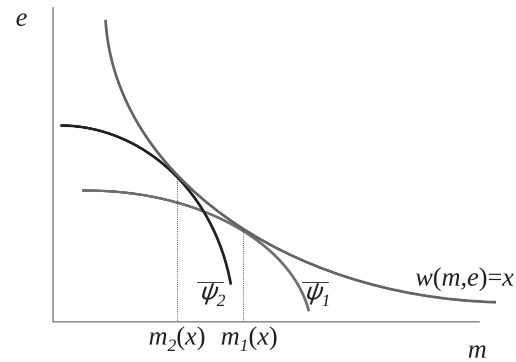

FIGURE 2. OPTIMAL MONETARY EXPENDITURE

This condition means that bidder 1's isocost curves are flatter. Together with (13), (14) implies

$$(15) \quad m_1(x) > m_2(x).$$

In words, bidder 1 relies more on monetary expenditures than does bidder 2, for any given bid. This is depicted in Figure 2.

The highest bid that bidder 2 can profitably make is again $\bar{x} = c_2^{-1}(v_2)$. Under the following condition, the cap binds only for bidder 1:

CONDITION 1: $m_2(\bar{x}) \le \bar{m} < m_1(\bar{x})$.

When Condition 1 holds, a cap of $\bar{m}$ will not affect bidder 2's bidding cost in equilibrium, but it will raise bidder 1's for x close to the supremum bid. As a consequence, the expected aggregate cost rises.

COROLLARY 3: *Imposition of a cap satisfying Condition* 1 *raises the expected aggregate cost.*

PROOF:

Since $\psi_1(m, e) \le \psi_2(m, e)$, we have $c_1(\cdot) \le c_2(\cdot)$ and $\bar{c}_1(\cdot) \le \bar{c}_2(\cdot)$. Further, $c_i(0) = \bar{c}_i(0) = 0$, and $c_i(\cdot)$ and $\bar{c}_i(\cdot)$ are continuous, strictly increasing, and unbounded. Hence, $(c_1(\cdot), c_2(\cdot))$ and $(\bar{c}_1(\cdot), \bar{c}_2(\cdot))$ are in C. Given Condition 1, $\bar{c}_2(x) = c_2(x)$ for $x \le \bar{x} = c_2^{-1}(v_2)$, and $\bar{c}_1(x) \ge c_1(x)$, with a strict inequality if $x > \hat{x}$, for some $\hat{x} < \bar{x}$. It follows that

$$(16) \quad \bar{c}_1(\bar{c}_2^{-1}(a)) = \bar{c}_1(c_2^{-1}(a)) \ge c_1(c_2^{-1}(a)), \ \forall a \le v_2,$$

with a strict inequality at $a = v_2$, so the shift from $(c_1(\cdot), c_2(\cdot))$ to $(\bar{c}_1(\cdot), \bar{c}_2(\cdot))$ is a strictly equalizing shift. The result then follows from Proposition 1.

The cap leaves bidder 2's cost function unchanged in the relevant range, but it raises bidder 1's over an interval. This tends to make bidder 2 more aggressive in the sense of raising her probability of winning.[11] Then, not only does the expected aggregate cost rise, but misallocation of the prize becomes more likely as well.

Condition 1 specifies an interval over which a cap has an equalizing effect; however, a cap is likely to have that effect in a broader set of circumstances. Even when a cap is binding for both bidders, (15) means that the cap will bind more tightly for bidder 1.

III. Conclusion

Kaplan and Wettstein (2006) have demonstrated that the analysis in Che and Gale (1998) depends on whether the cap on lobbying is rigidly enforced. We have shown that CG's results do not depend on that feature. Given asymmetric costs of lobbying, their results can hold even when the cap has a continuous effect. The analysis here has identified circumstances under which a cap levels the playing field for lobbyists with asymmetric costs. In such circumstances, a cap may lead to increased aggregate expenditure and a less efficient allocation. More generally, regulatory interventions that affect costs differentially may have these effects.

[11] The probability rises given the reasonable assumption that $\partial\psi_1/\partial e$ rises and $\partial w/\partial e$ falls as one moves along the respective isocost and isobid curves, in the direction of higher e. To see this, first compute the probability that bidder 2 wins under $(c_1(\cdot), c_2(\cdot))$:

$$\int_0^{c_2^{-1}(v_2)} F_1(x)\, dF_2(x) = \left(\frac{1}{v_1 v_2}\right) \int_0^{v_2} a\left(\frac{c_1'(c_2^{-1}(a))}{c_2'(c_2^{-1}(a))}\right) da.$$

Hence, the result will hold if

$$\frac{\bar{c}_1'(\bar{c}_2^{-1}(a))}{\bar{c}_2'(\bar{c}_2^{-1}(a))} \geq [>] \frac{c_1'(c_2^{-1}(a))}{c_2'(c_2^{-1}(a))} \text{ for all (for a positive measure of) } a \in [0, v_2].$$

Since $\bar{c}_2(\cdot) = c_2(\cdot)$ in the relevant region, this condition boils down to

$$\bar{c}_1'(x) \geq [>]\, c_1'(x) \text{ for all (for a positive measure of) } x \in [0, c_2^{-1}(v_2)].$$

This result holds given the assumption above since

$$\bar{c}_1'(x) = \bar{\mu} = \frac{\partial\psi_1(\hat{m}, \hat{e})/\partial e}{\partial w(\hat{m}, \hat{e})/\partial e} \geq [>]\, \mu = \frac{\partial\psi_1(m, e)/\partial e}{\partial w(m, e)/\partial e} = c_1'(x),$$

where $\bar{\mu}$ and μ are the multipliers on $x = w(m, e)$ in (12) when $i = 1$, with and without a cap, respectively; and $(\hat{m}, \hat{e})$ and (m, e) are the minimizers, with and without a cap, respectively. The inequality follows from the assumption above since $\hat{m} \leq [<]\, m$, $\hat{e} \geq [>]\, e$, and $w(\hat{m}, \hat{e}) = x = w(m, e)$.

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