Keeping the Listener Engaged: a Dynamic Model of Bayesian Persuasion

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Bayesian Persuasion

- **Classical question:** How (much) can a sender persuade a rational receiver to take a particular action? (e.g., *seller-buyer, media-voters, prosecutor-judge, entrepreneur-investor.....*)
- **An important assumption:** Commitment, achieved by *instantaneous* and *unrestricted* experimentation. We relax the commitment power with a model that has:
- **Main features:**
  - Persuasion takes time and cost: Information takes real time to generate/convey; costly for the sender to generate and for the receiver to process.
  - No commitment to future actions: Sender cannot commit to future experiments/persuasion.
- **Questions:**
  - Is dynamic persuasion possible? What payoffs can be achieved?
  - Behavioral implications: Dynamic choice of information structures

Model

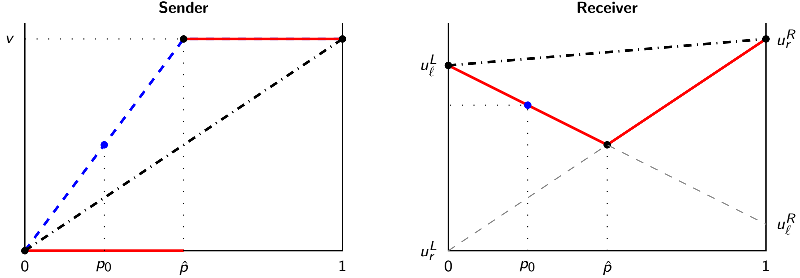
- **Two States:** $\omega \in \{L, R\}$
- **Receiver:**
  - chooses a binary action $a \in \{\ell, r\}$
  - prefers to "match" the state: $u_\ell^L > u_r^L$, $u_r^R > u_\ell^R$, where $\boldsymbol{u_a^\omega}$ is payoff from a in state ω.
  - Notation:

$$U_a(p) = pu_a^R + (1-p)u_a^L, \quad a \in \{\ell, r\}$$
$$\mathcal{U}(p) = \max\{U_r(p), U_\ell(p)\} > 0$$

- **Sender:**
  - receives state-independent payoff $v \cdot \mathbf{1}_{\{a=r\}}$
  - performs experiments over time to "persuade" receiver.

Static Benchmark: Kamenica-Gentzkow Model (graphical)

- Sender picks an arbitrary Blackwell experiment.
- Let $\hat{p}$ be such that $U_\ell(\hat{p}) = U_r(\hat{p})$. Suppose prior is $p_0 < \hat{p}$.
- **Solution:** Sender maximizes the prob of inducing posterior $\geq \hat{p}$ $\Rightarrow$ two beliefs 0 and $\hat{p}$.



Observations

- *R*-signal sent excessively compared to full information.
- "Fully-revealing of *L*" in case of *L*-signal
- **The receiver enjoys no rents.**

Our Model: Dynamic Extension

- Continuous time, infinite time horizon.

Timing

At each point $t \geq 0$ in time,

1. Sender picks an experiment (to be described later) at flow cost $c > 0$ or "passes" ($=$ null information)
2. Receiver observes the *experiment* and its *outcome*, and either **takes an game-ending action** $a \in \{\ell, r\}$, or **waits**.
   - If the receiver waits and listens to an experiment he incurs flow cost $c > 0$.
   - No cost is incurred if the sender "passes."

Feasible Experiments: General Poisson Models

- At each point, S chooses a mix of targeted Poisson experiments $i \in I$ with (fractional) units α_i, $\sum_i \alpha_i \leq 1$.

- Each Poisson experiment i generates a signal that arrives with rates $\lambda^L := \nu^L + \mu$ and $\lambda^R := \nu^R + \mu$ in states L and R such that $\nu^L + \nu^R \leq \lambda$, for some $\lambda > 0$, $\mu \geq 0$.

- Effectively two indistinguishable signals:
  - Real signal: with arrival rates ν^L and ν^R in states L and R, where $\nu^L + \nu^R \leq \lambda$, for some $\lambda > 0$ ("info bound")
  - Noise ("inflation"): with the same arrival rate μ in each state.

- Sender mixes across $(\nu_i^L + \mu_i, \nu_i^R + \mu_i)$ with weights $\alpha_i \Rightarrow$ arrives at rates $\alpha_i(\lambda_L + \mu, \lambda_R + \mu)$.

Illustration of a feasibele experiment

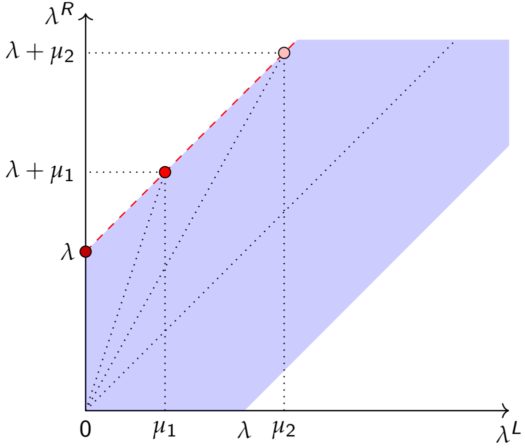


Figure: Arrival rates of feasible Poisson experiments.

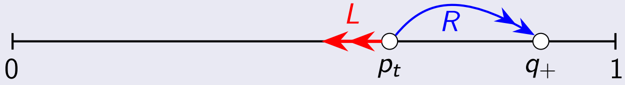
Posterior beliefs induced by Poisson jump

- S can choose a feasible (λ^L, λ^R) so that, for any current belief p, a breakthrough signal induces "any" posterior belief q arriving at rate $\frac{p(1-p)}{|q-p|}\lambda$.
  - Nests *conclusive good news* or *conclusive bad news*: Set $q = 1$ or 0.
  - Allows for any directionality ("good" news $q > p$ or "bad" news $q < p$) and any degree of accuracy (q can be far from or close to p), and can mix different Poisson experiments.
- **Important feature**: *Real information takes time; the more precise, the longer it takes.*

Several experiments

L-drifting experiment (with right-jumps $q_+ > p$)

- R-signals: belief jumps to q_+ at arrival rate of $\frac{p(1-p)}{|q_+ - p|}\lambda$
- L-signals: belief drifts to the left at rate $\dot{p}_t = -\lambda p(1-p)$

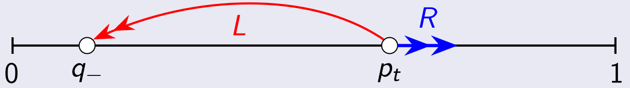


- Sender may choose the "precision" of R-evidence.
  - For example: can target $q_+ = \hat{p}$.
- **Tradeoff**: More precise signals are slower to generate.

Several experiments

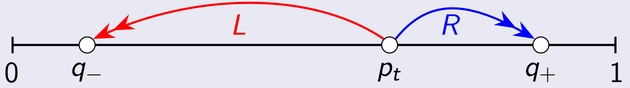
R-drifting experiment (with left-jumps to $q_- < p$):

- L-signals: belief jumps to q_- at rate $\frac{p(1-p)}{|q_- - p|}\lambda$
- R-signals: belief drifts toward right at rate $\dot{p}_t = \lambda p(1-p)$



"Stationary" Experiment

- Splitting attention ($\alpha = 1/2$), we obtain **2 jumps and no drift**
- Jumps to q_- and q_+ at rates $\frac{\lambda p(1-p)}{2|q_\bullet - p|}$,—no drift.



Our Model: Dynamic Extension

Equilibrium

- **Markov Perfect equilibria (MPE)**: Subgame perfect equilibrium where strategies depend only on the payoff-relevant state p, regardless of the history.
- **Additional credibility restriction:** The MPE should be a limit of discrete time game equilibria—Sender optimizes even on experiments succeeding with vanishing probability.

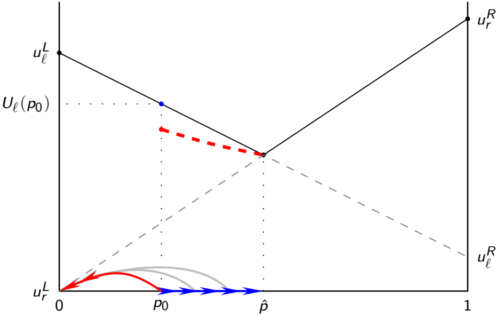
Literature

- Bayesian Persuasion: Kamenica and Gentzkow (2011,...), ..., Aumann/Maschler (1995)
- Wald Decision: Wald (1947), Arrow, Blackwell, and Girshick (1949), Moscarini and Smith (2001), Che and Mierendorff (2018), Nikandrova and Pancs (2018), Mayskaya (2017), Zhong (2018), Henry and Ottaviani (2019), McClellan (2017)
- Dynamic Persuasion: Brocas and Carrillo (2007), Kremer, Mansour and Perry (2014), Au (2015), Ely (2017), Renault, Solan and Vieille (2017), Bizzoto, Rudiger and Vigier (2017), Che and Hörner (2018), Henry and Ottaviani (2019), Ely and Szydlowski (2020), Orlov, Skrzypacz and Zryumov (2020).
- Repeated Persuasion/Communication: Margaria and Smolin (2018), Best and Quigley (2017), Mathevet, Pearce, and Stachetti (2018).

  Difference: Permanent state, MPE, slow learning.

Dynamic Implementation of Optimal Static Experiment

- Fix $p_0 < \hat{p}$.
- Can replicate KG: dynamic experiment that leads to beliefs 0 and $\hat{p}$
- For example: R-drifting experiment until belief reaches $\hat{p}$.

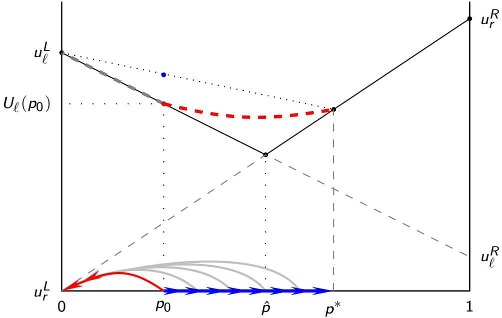


- **Problem:** Receiver does not wait if she does not get rent that compensates for flow cost.

$\Rightarrow$ **KG experiment can't persuade receiver to listen.**

Fix: Dynamic Commitment

- *Solves the problem if Sender can commit to future experiments*
  - Example: Commit to R-drifting until the belief reaches $p^* > \hat{p}$.



- Similar to KG except for provision of "rents" to compensate for Receiver's flow cost. Can approximate KG if $c \to 0$.
- *But will this work without commitment?*

Is persuasion possible without commitment?

- **No**
  - There is an MPE with total persuasion failure regardless of $c > 0$.

- **Yes**
  - For each $p_0 < \hat{p}$, some dynamic commitment can be supported as MPE if c is low enough.
  - As $c \to 0$, a KG experiment as well as full revelation (and anything in between) is dynamically credible. $\Rightarrow$ **Folk Theorem**

MPE: Persuasion Failure

Theorem (Persuasion Failure MPE)

For any $c > 0$, there exists a MPE in which no persuasion occurs.

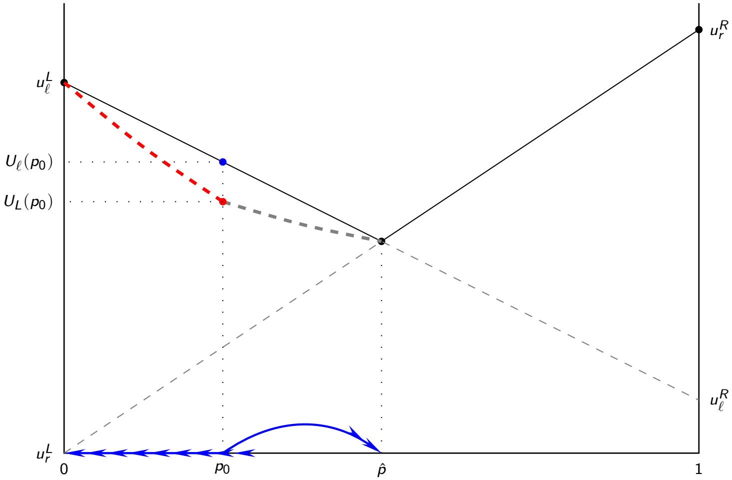
Proof.

MPE strategy profile:

- Receiver never waits—he picks r if $p \geq \hat{p}$ and ℓ for $p < \hat{p}$.
- Sender "passes" if $p \geq \hat{p}$ (or if $p < \hat{p}$ is very low), and performs an L-drifting with jump to $\hat{p}$ if $p < \hat{p}$.

□

Persuasion failure: illustration



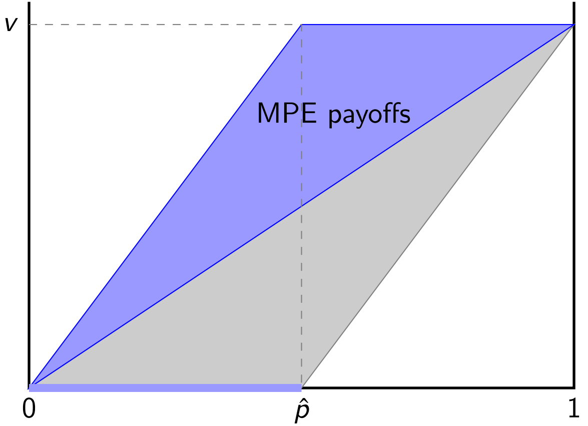
Persuasion MPE: Folk Theorem

More surprisingly, persuasion is possible in MPE. In fact, we can establish a folk theorem.

Theorem (Folk theorem)

Any sender payoff between KG benchmark and "full revelation" is supported in an MPE for any c sufficiently small.

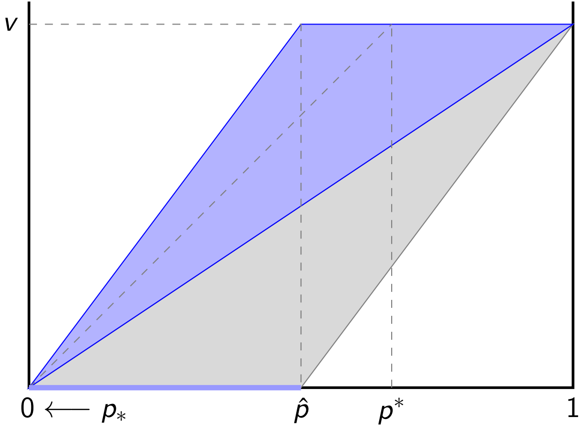
Persuasion MPE: Folk Theorem — Sender's Payoff Set



as $c \to 0$.

Constructing persuasion equilibria

We construct an MPE in which: S persuades and R waits if $p \in [p_*, p^*]$.



- Dashed line: Equilibrium payoffs for fixed p^* as $c \to 0$
- Can choose $p^* \searrow \hat{p}$ as $c \to 0$

Illustration of Persuasion Equilibria

- The construction of persuasion equilibria depend on whether

$$(C1) \qquad p^* < \eta, \text{ where } \eta = .943$$

— *how demanding persuasion target p^* is.*

$$(C2) \qquad v > U_r(p^*) - U_\ell(p^*).$$

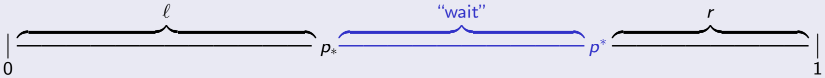
— *relative incentive for S to persuade vs for R to listen.*

MPE under $(C1)$ and $(C2)$

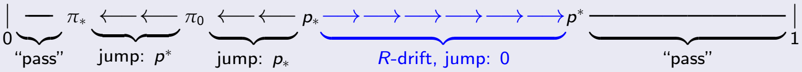
- Given $(C1)$ and $(C2)$, for $c > 0$ sufficiently small, there exists a persuasion MPE with persuasion target p^*:

Persuasion MPE

Receiver's strategy:



Sender's strategy:



- At p_*, R is indifferent to ℓ and "wait."
- May approximate KG: Can choose $p^* \to \hat{p}$ and $p_* \to 0$ as $c \to 0$.

Intuition: *Power of Beliefs*

- Why is Sender continuing to experiment even after reaching $\hat{p}$? Why not stop at $\hat{p}$

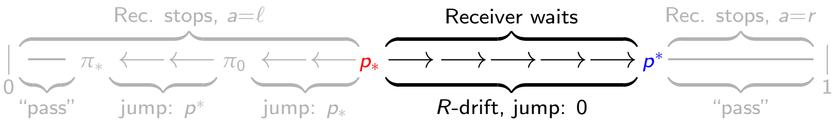
- Suppose Sender stopped at $\hat{p}$ (i.e., "deviated"). $\Rightarrow$ Receiver would never choose r but rather wait.

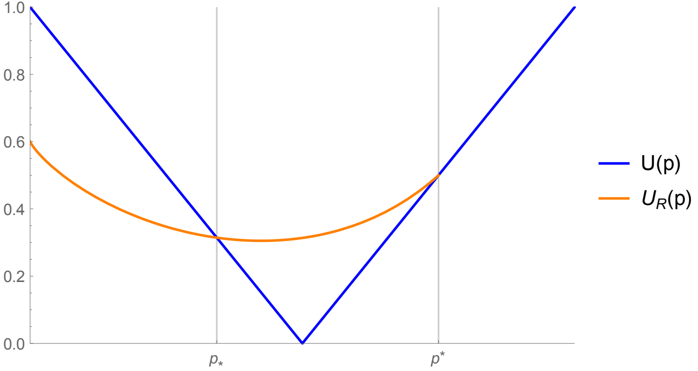
- Why? Why is Receiver waiting even after $\hat{p}$ is reached?

$\Rightarrow$ Because, if Receiver waits, Sender will continue experimenting.

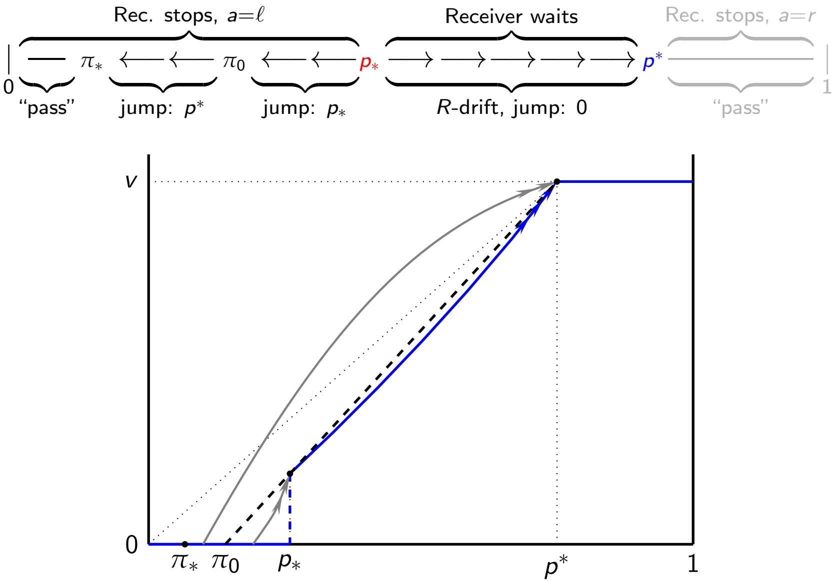
  - Power of equilibrium beliefs: reminiscent of Che and Sákovics, ECMA, 2004.

Receiver Incentive

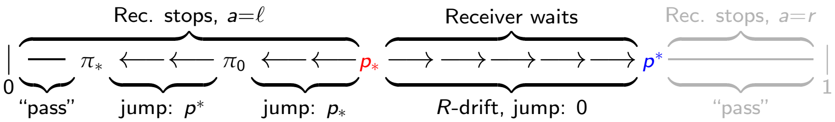




Sender Incentive



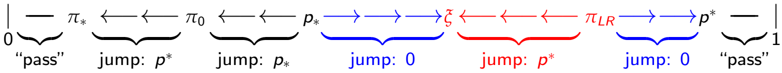
Dynamics of Persuasion



- When Receiver is already interested in listening (i.e., $p \in (p_*, p^*)$):
  - $\Rightarrow$ Confindence building; tries to rule out L
  - $\Rightarrow$ Persuasion backloaded.
- When Receiver is skeptical (i.e., $p < p_*$):
  - $\Rightarrow$ Sender throws a "Hail Mary"
  - $\Rightarrow$ Persuasion almost surely fails.

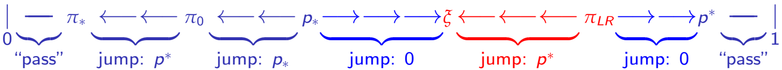
The case of: $p^* > \eta$

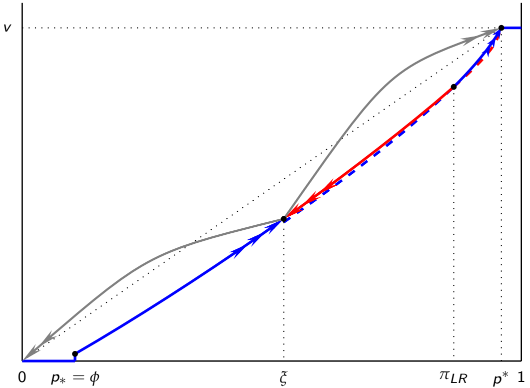
- still assume $(C2): v > U_r(p^*) - U_\ell(p^*)$)
- For $c > 0$ small, an MPE looks like:



  - At ξ: stationary with jumps to $q_- = 0$ and $q_+ = p^*$.
  - Alternative dynamic strategies lead to the same posterior distr supported on $\{0, p^*\}$.
  - *But they differ in expected persuasion costs*.

Approximating Full Revelation

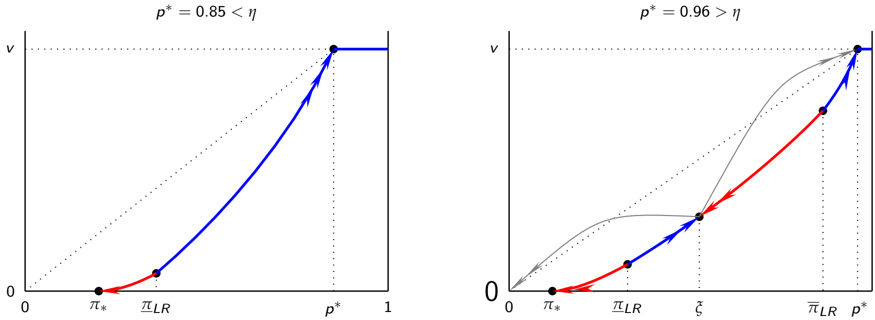




- $p_* \to 0$ as $c \to 0$
- $\pi_{LR} \to 1$ and $\xi \to 1/2$ as $p^* \to 1$.

The case of $\neg(C2): v < U_r(p^*) - U_\ell(p^*)$

Sender's strategies and values:



- For a low $p > p_*$, the sender uses L-drifting—"confidence spending." Similar to "Hail Mary," but on path here.
- Posteriors supported on $\{0, \pi_*, p^*\}$.

Summary: Main Contributions

1. Introduce sequential information production into Bayesian Persuasion model:
   - Relax commitment power.
   - Power of beliefs allows to sustain persuasion.

2. Folk Theorem yields large set of equilibrium outcomes:
   - No persuasion, and any outcome between KG and full revelation can arise.

3. Characterize Persuasion Dynamics.
   - Building confidence vs. spending confidence.
   - Persuasion dynamics depend on type of equilibrium.

4. Tractable model of dynamic strategic information choice.

Thank you!